## Markets for Risk Management

## Insurance Economics

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## Professor Information

- I hold appointments at Baylor University as the Frank S. Groner Memorial Chair of Finance and Professor of Finance and Insurance, and at the Wharton School as a Visiting Scholar.
- Previous faculty appointments at Pennsylvania State University, University of Texas-Austin, and Louisiana State University Business Schools


## Professor Information

- Research: corporate risk management and "insurance finance"
- Service:
- Editorial Boards: Geneva Risk and Insurance Review and Journal of Risk and Insurance
- Academic Associations:
- President of American Risk and Insurance Association (ARIA).
- Past President of Risk Theory Society (RTS)


## Convergence of Finance and Insurance

- Convergence between finance and risk management \& insurance has been occurring for some time now, in practice as well as in theory.
- As The Economist has famously noted, "The business of financing companies is converging with the business of insuring them."


## Convergence of Finance and Insurance

- Loubergé (2013) notes that the development of risk and insurance economics has involved "...the applications of new financial paradigms, such as contingent claims analysis, to the analysis of insurance firms, insurance markets, and corporate risk management, a development which links more closely insurance economics to
financial economics, and insurance to finance."


## Topics Covered in this Course

- Insurance Economics (Lecture 1)
- Financial Pricing Models and Capital Allocation (Lectures 2-3)
- Corporate Risk Management (Lecture 4)
- Reinsurance (Lecture 5)


## Why a lecture on insurance economics?

- Insurance economics provides a set of models that help us to better understand behavior towards risk; e.g.,
- How do alternative contract designs (involving different combinations of linear and non-linear payoffs) affect the pricing and sharing of risk?
- How does risk sharing affect incentives?
- Although many examples focus on insurance problems, the underlying principles have broad application outside of insurance per se.


## Topics Covered in Lecture 1

- Insurance supply: risk pooling, risk spreading, and risk transfer mechanisms
- A simple "single risk" model of the demand for insurance
- Endogenous "background" risks
- Moral Hazard
- Adverse Selection


## For Future Reference!

- Schlesinger, H., 2013, "The Theory of Insurance Demand," Chapter 7 in G. Dionne, editor, Handbook of Insurance (2 $2^{\text {nd }}$ edition, Boston: Kluwer Academic Publishers).
- Winter, R. A., 2013, "Optimal Insurance Contracts Under Moral Hazard," Chapter 9 in G. Dionne, editor, Handbook of Insurance (2 ${ }^{\text {nd }}$ edition, Boston: Kluwer Academic Publishers).
- Dionne, G., Fombaron, N., and N. Doherty, 2013, "Adverse Selection in Insurance Contracting," Chapter 10 in G. Dionne, editor, Handbook of Insurance (2 ${ }^{\text {nd }}$ edition, Boston: Kluwer Academic Publishers).


## Risk Pooling

Let $E\left(L_{i}\right)=$ expected loss for insured $i$ and $E\left(L_{T}\right)=\sum_{i=1}^{n} E\left(L_{i}\right)=$ total expected loss of the risk pool. Then

$$
\begin{equation*}
E\left(L_{p}\right)=\sum_{i=1}^{n} w_{i} E\left(L_{i}\right) \tag{1}
\end{equation*}
$$

$=$ average loss per policy,
where $w_{i}=E\left(L_{i}\right) / E\left(L_{T}\right)$. Similarly,

$$
\begin{equation*}
\sigma_{L p}^{2}=\sum_{i=1}^{n} \sum_{j=1}^{n} w_{i} w_{j} \rho_{i j} \sigma_{i} \sigma_{j} \tag{2}
\end{equation*}
$$

$=$ average risk per policy, where
$\rho_{i j}=\sigma_{i} / \sigma_{i} \sigma_{j}=$ correlation between losses on policy $i$ and policy $j$.

## Risk Pooling

- Let losses be identically distributed; i.e., $E\left(L_{i}\right)=\mu, \sigma_{t}^{2}=\sigma^{2}$, and $w_{i}=1 / n$ for all insureds, while $\rho_{i j} \sigma_{i} \sigma_{j}=\left\{\begin{array}{l}\rho \sigma^{2} \text { if } i \neq j \\ \sigma^{2} \text { if } i=j\end{array}\right.$. Therefore, (1) and (2) are rewritten

$$
\begin{gather*}
E\left(L_{p}\right)=\sum_{i=1}^{n} w_{i} E\left(L_{i}\right)=(1 / n) n \mu=\mu, \text { and }  \tag{1a}\\
\\
\sigma_{L_{p}}^{2}=\frac{1}{n^{2}} \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i j} \sigma^{2}  \tag{2a}\\
=\frac{\sigma^{2}}{n}+\frac{n-1}{n} \rho \sigma^{2} .
\end{gather*}
$$

## Risk Pooling

- Since $\sigma_{L_{p}}^{2}=\frac{\sigma^{2}}{n}+\frac{n-1}{n} \rho \sigma^{2}, \lim _{n \rightarrow \infty} \sigma_{L_{p}}^{2}=\rho \sigma^{2}$; i.e., only covariance risk remains.
- Now suppose losses are $i i d$. Then $\lim _{n \rightarrow \infty} \sigma_{L_{\rho}}^{2}=0$. Thus the average loss becomes more predictable as the number of risks pooled becomes large.
- By pooling many independent risks, insurers can treat uncertain losses as almost known.
$\circ$ Risk pooling effectively defrays risk by exploiting the law of large numbers.


## Risk Pooling and Insolvency

- Assume that losses are normal and iid, where total $\operatorname{losses} L_{\mathrm{T}}=\sum_{i=1}^{n} L_{i}, E\left(L_{\mathrm{T}}\right)=\sum_{i=1}^{n} E\left(L_{i}\right)=n \mu$ and $\sigma_{L_{T}}^{2}=n \sigma^{2}$.
- Suppose we want the pool to be able to pay losses with some level of certainty; i.e., $\operatorname{Pr}\left(L_{T}<n \mu+S\right)=p$, where $p$ represents our threshold insolvency probability and $S$ represents the insurer's surplus.
- Standardizing the normal random variable, we obtain

$$
\operatorname{Pr}\left(\frac{L_{T}-n \mu}{\sigma_{L_{T}}}=z<\frac{S}{\sigma_{L_{T}}}=\lambda\right)=p .
$$

## Risk Pooling and Pricing



- The level of surplus $S$ that will produce a solvency rate of $p$ is $S=\lambda \sigma_{L_{T}}$.
- Total risk pool premium is $P_{T}=n \mu+\lambda \sigma_{L_{T}}=n \mu+\lambda \sqrt{n} \sigma$.
- $\therefore$ the premium per policyholder is $P_{i}=\underset{\text { pure premium }}{\mu}+\frac{\lambda \sigma}{\sqrt{n}}$.


## Risk Spreading

- Risk pooling "breaks down" when risks are not
independent; note that $\lim _{n \rightarrow \infty} \frac{\sigma^{2}}{n}+\frac{n-1}{n} \rho \sigma^{2}=\rho \sigma^{2}$; if $\rho=1$, then $\sigma_{L_{p}}^{2}=\sigma^{2}$.
- Catastrophic risks come to mind (e.g., earthquakes, floods, cyclones, RBNC terrorist events, nuclear war, etc.) such risks are (to some extent) nondiversifiable.

O This helps to explain some of the "fine print" in insurance contracts; e.g., catastrophe exclusions.

## Risk Spreading

- Catastrophe risks are often managed by some combination of ex ante and expost risk spreading by government.
- In theory, risk spreading may improve social welfare, even though the total amount of risk is not diminished.
- However, this is Pareto inefficient, since it is not possible to improve catastrophe victims' welfare without reducing the welfare of non-victims.
- Recent innovations in alternative risk transfer (e.g., act of God bonds, terrorism bonds, etc.) represent private sector attempts to manage catastrophe risk using capital market instruments.


## The Demand for Insurance

- An individual has wealth $W_{0}$ and will suffer a loss $L$ with probability $\pi$. Thus she owns the lottery ( $\left\langle W_{0}-L\right.$, $\left.\left.W_{0}\right\rangle,\langle\pi, 1-\pi\rangle\right)$.
- She can take out insurance, in which case she must pay a premium $P=p C$, where $p$ is the premium rate and $C$ is the level of coverage.
- Thus this individual may exchange the lottery she owns for the "insurance" lottery

$$
\left(\left\langle W_{0}-p C-L+C, W_{0}-p C\right\rangle,\langle\pi, 1-\pi\rangle\right) .
$$

## The Demand for Insurance

- Consider a special case of the insurance lottery, where $C$ $=L$; i.e., risk is fully insured. With full insurance, state contingent wealth is $W_{0}-p C$ regardless of whether a state contingent loss occurs; thus under full insurance, she (the consumer) exchanges an uncertain loss ( $L$ ) for a certain loss ( $p C$ ).
- She will buy insurance only if a $C$ exists such that the expected utility of being insured exceeds the expected utility of remaining uninsured; i.e.,

$$
\begin{gathered}
\pi U\left(W_{0}-p C-L+C\right)+(1-\pi) U\left(W_{0}-p C\right)> \\
\pi U\left(W_{0}-L\right)+(1-\pi) U\left(W_{0}\right) .
\end{gathered}
$$

## The Demand for Insurance

- The optimal level of insurance coverage is determined by maximizing expected utility; i.e., by finding the value of $C$ at which the following equation is maximized:

$$
E(U(W))=\pi U\left(W_{0}-L+(1-p) C\right)+(1-\pi) U\left(W_{0}-p C\right)
$$

- In order to maximize expected utility, we must solve the first order condition:

$$
\pi(1-p) U^{\prime}\left(W_{0}-L+(1-p) C\right)=p(1-\pi) U^{\prime}\left(W_{0}-p C\right) .
$$

- Bernoulli principle: Suppose $p=\pi$. Then

$$
\begin{aligned}
& U^{\prime}\left(W_{0}-p C-L+C\right)=U^{\prime}\left(W_{0}-p C\right) \\
& \Rightarrow W_{0}-p C-L+C=W_{0}-p C
\end{aligned}
$$

i.e., if the insurance premium is actuarially fair, then full coverage $(C=L)$ is optimal.

## The Demand for Insurance

- Now suppose $U(W)=\ln W$.
- The first order condition implies that

$$
\frac{\pi(1-p)}{W_{0}-L+(1-p) C}=\frac{p(1-\pi)}{W_{0}-p C} .
$$

- Solving for $C$, we find that

$$
C=\frac{(\pi-1) p L+(p-\pi) W_{0}}{p(p-1)} .
$$

- Bernoulli Principle: Suppose $p=\pi$, i.e., insurance is actuarially fair. Then $C=\frac{L p(p-1)}{p(p-1)}=L$; i.e., it is optimal to fully insure.


## Effect of Changes in Initial Wealth

- First, consider the effect of changes in initial wealth on the demand for insurance; this is analyzed by differentiating $C$ $=[p(p-1)]^{-1}\left(L p(\pi-1)+W_{0}(p-\pi)\right)$ with respect to $W_{0}$, resulting in the following expression:

$$
\frac{\partial C}{\partial W_{0}}=\underbrace{[p(p-1)]^{-1}}_{-} \underbrace{(p-\pi)}_{0,+} \leq 0 .
$$

- If insurance is actuarially fair; i.e., $p=\pi$, then $\partial C / \partial W_{0}=0$.
- To be expected, since $p=\pi$ implies full coverage ( $C=L$ ) irrespective of one's level of initial wealth $W_{0}$.
- If insurance is unfair; i.e., $p>\pi$, then there is an inverse relationship between $C$ and $W_{0}$, since the product of a negative first term multiplied by a positive second term is negative.


## Effect of Changes in Loss Frequency

- Next, we study the effect that a change in loss frequency has upon the optimal value for $C$; this is analyzed by differentiating $C=[p(p-1)]^{-1}\left(L p(\pi-1)+W_{0}(p-\pi)\right)$ with respect to $\pi$ :

$$
\frac{\partial C}{\partial \pi}=\underbrace{[p(p-1)]^{-1}}_{-} \underbrace{\left(L p-W_{0}\right)}_{-}>0
$$

- Since the consumer cannot spend more than initial wealth on insurance, $L p-W_{0}<0$.
- Consequently, there is a positive relationship between $C$ and $\pi$, i.e., the demand for insurance is higher, the higher the loss frequency.


## Effect of Changes in Loss Severity

- Next, we study the effect that a change in loss severity has upon the optimal value for $C$ by differentiating $C$ $=[p(p-1)]^{-1}\left(L p(\pi-1)+W_{0}(p-\pi)\right)$ with respect to $L$ :

$$
\frac{\partial C}{\partial L}=[p(p-1)]^{-1}(p(\pi-1))=\underbrace{(p-1)^{-1}}_{-} \underbrace{(\pi-1)}_{-}>0 .
$$

- Consequently, there is a positive relationship between $C$ and $L$; i.e., the demand for insurance is higher, the higher the accident severity.


## Effect of Changes in the Premium

- Next, we study how a change in the insurance premium affects the optimal value for $C$; this is analyzed by differentiating
$C=[p(p-1)]^{-1}\left(L p(\pi-1)+W_{0}(p-\pi)\right)$ with respect to $p:$

$$
\frac{\partial C}{\partial p}=\frac{2 p \pi W_{0}-\pi\left(L p^{2}+W_{0}\right)-p^{2}\left(W_{0}-L\right)}{(p-1)^{2} p^{2}}
$$

- A priori, we expect that the demand for insurance will be inversely related to the insurance premium.
- Clearly, the denominator is positive.
- In the numerator, the first term is positive whereas the second and third terms are both negative.
- Hoy and Robson (1981 Economics Letters) have shown that insurance cannot be Giffen if the coefficient of relative risk aversion is $\leq 1$; therefore, $\partial C / \partial p<0$.


## Important Insurance Theorems

- Mossin's Theorem: If proportional insurance is available for an actuarially fair (unfair) premium, then full (partial) coverage is optimal.
- Arrow's Theorem: Other things (i.e., premium and expected indemnity value) equal, risk-averse agents prefer insurance policies with deductibles over all other contract forms.


## Mossin's Theorem

- Suppose initial wealth $\left(W_{0}\right)$ is $\$ 120, \$ 100$ of which is invested in an asset that has a $25 \%$ probability of being destroyed by fire. $U(W)=W^{5}$, and the premium for a "full coverage" insurance policy is $\$ 25$.
- $25 \%$ of the time, $W_{\mathrm{s}}=W_{0}-\alpha \mathrm{P}^{\mathrm{i}}-(1-\alpha) L$, where $\alpha$ is the coinsurance rate and $P^{i}$ is the price of full insurance coverage. Thus $W_{s}=120-\alpha 25-(1-\alpha) 100$ $=20+75 \alpha$.
- $75 \%$ of the time, $W_{s}=W_{0}-\alpha P^{i}=120-\alpha 25$.


## Mossin's Theorem

Expected utility is $E(U(W))=.25(20+75 \alpha)^{.5}+.75(120-$ $\alpha 25)^{5}$. The optimal value for $\alpha$ maximizes expected utility; therefore,
$\frac{d E(U(W))}{d \alpha}=9.375(20+75 \alpha)^{-.5}-9.375(120-25 \alpha)^{-.5}=0$.

$$
\begin{aligned}
& \therefore(20+75 \alpha)^{-.5}=(120-25 \alpha)^{-.5} \\
& \therefore 20+75 \alpha=120-25 \alpha \\
& \therefore 100 \alpha=100 \rightarrow \alpha=1 .
\end{aligned}
$$

In other words, full coverage is optimal when insurance is actuarially fair.

## Mossin's Theorem

- Next, suppose that the price of a full coverage policy is $\$ 40$. Calculate the optimal value for $\alpha$.

This price change implies that $E(U(W))=.25(20+$ $60 \alpha)^{5}+.75(120-\alpha 40)^{5}$; therefore,

$$
\frac{d E(U(W))}{d \alpha}=7.5(20+60 \alpha)^{-.5}-15(120-40 \alpha)^{-.5}=0 .
$$

$$
\therefore 7.5(120-40 \alpha)^{.5}=15(20+60 \alpha)^{.5}
$$

$$
\therefore 56.25(120-40 \alpha)=225(20+60 \alpha)
$$

$$
\therefore 6,750-2,250 \alpha=4,500+13,500 \alpha
$$

$$
\therefore 2,250=15,750 \alpha \rightarrow \alpha=1 / 7
$$

## Mossin's Theorem



## Arrow's Theorem

- "If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100 per cent coverage above a deductible minimum."


## Arrow's Theorem

- Proof: Consider the following insurance policies:
- A "deductible" policy with deductible $d$. In the event of a claim, the indemnity $I(L, d)$ is of the form $I(L, d)=$ $\operatorname{Max}(L-d, 0)$; and
- A "general" policy which pays indemnity $I(L)$, where $0 \leq I(L) \leq L$.
- The deductible policy will be preferred to the general policy if the following condition holds:

$$
\begin{align*}
& E\left(U\left(W_{0}-L+I(L)\right)\right) \\
& \quad \leq E\left(U\left(W_{0}-L+\operatorname{Max}(L-d, 0)\right)\right) \tag{12}
\end{align*}
$$

## Arrow's Theorem

Note that a concave curve is located below its tangents; i.e.,

$$
\begin{equation*}
U(y) \leq U(x)+U^{\prime}(x)(y-x) \tag{13}
\end{equation*}
$$



Figure 1. Graphical Illustration of Inequality (13).

## Arrow's Theorem

- Let $y=W_{0}-L+I(L)$ and
$x=W_{0}-L+\operatorname{Max}(L-d, 0)$. Then (13) implies (14):

$$
\begin{align*}
U\left(W_{0}-L\right. & +I(L)) \leq U\left(W_{0}-L+\operatorname{Max}(L-d, 0)\right) \\
& +U^{\prime}(x)(I(L)-\operatorname{Max}(L-d, 0)) \tag{14}
\end{align*}
$$

- Since both policies have the same actuarial value, this implies $E(I(L)-\operatorname{Max}(L-d, 0))=0$; thus $E\left(U\left(W_{0}-L+I(L)\right)\right) \leq E\left(U\left(W_{0}-L+\operatorname{Max}(L-d, 0)\right)\right)$.


## Insurance Indemnities \& Premiums

|  |  |  |  |  |  | Self |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Full | Deductible Coinsurance |  | Limit |  |
| $\mathrm{p}(\mathrm{s})$ | $\mathrm{L}(\mathrm{s})$ | Insurance | Insurance | $\$ 20$ | $75 \%$ | $\$ 100$ |
| $50 \%$ | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ |
| $10 \%$ | $\$ 20$ | $\$ 0$ | $\$ 20$ | $\$ 0$ | $\$ 15$ | $\$ 20$ |
| $20 \%$ | $\$ 40$ | $\$ 0$ | $\$ 40$ | $\$ 20$ | $\$ 30$ | $\$ 40$ |
| $10 \%$ | $\$ 100$ | $\$ 0$ | $\$ 100$ | $\$ 80$ | $\$ 75$ | $\$ 100$ |
| $10 \%$ | $\$ 200$ | $\$ 0$ | $\$ 200$ | $\$ 180$ | $\$ 150$ | $\$ 100$ |
|  |  |  |  |  |  |  |
| $\mathrm{E}(\cdot)$ | $\$ 40$ | $\$ 0$ | $\$ 40$ | $\$ 30$ | $\$ 30$ | $\$ 30$ |
| Premium |  | $\$ 0$ | $\$ 48$ | $\$ 36$ | $\$ 36$ | $\$ 36$ |

## Final Wealth Distributions

Upper
Self Full Deductible Coinsurance Limit

| $\mathrm{p}(\mathrm{s})$ | $\mathrm{L}(\mathrm{s})$ | Insurance | Insurance | $\$ 20$ | $75 \%$ | $\$ 100$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $50 \%$ | $\$ 0$ | $\$ 260$ | $\$ 212$ | $\$ 224$ | $\$ 224$ | $\$ 224$ |
| $10 \%$ | $\$ 20$ | $\$ 240$ | $\$ 212$ | $\$ 204$ | $\$ 219$ | $\$ 224$ |
| $20 \%$ | $\$ 40$ | $\$ 220$ | $\$ 212$ | $\$ 204$ | $\$ 214$ | $\$ 224$ |
| $10 \%$ | $\$ 100$ | $\$ 160$ | $\$ 212$ | $\$ 204$ | $\$ 199$ | $\$ 224$ |
| $10 \%$ | $\$ 200$ | $\$ 60$ | $\$ 212$ | $\$ 204$ | $\$ 174$ | $\$ 124$ |
|  |  |  |  |  |  |  |
| $\mathrm{E}(\cdot)$ | $\$ 40$ | $\$ 220$ | $\$ 212$ | $\$ 214$ | $\$ 214$ | $\$ 214$ |
| $\sigma$ | 61.32 | 61.32 | 0.00 | 10.00 | 15.33 | 30.00 |

## Expected Utility Calculations

Upper

|  |  | Self | Full | Deductible Coinsurance |  | Limit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}(\mathrm{s})$ | $\mathrm{L}(\mathrm{s})$ | Insurance Insurance |  | $\$ 20$ | $75 \%$ | $\$ 100$ |
| 0.50 | 0.00 | 16.1245 | 14.5602 | 14.9666 | 14.9666 | 14.9666 |
| 0.10 | 20.00 | 15.4919 | 14.5602 | 14.2829 | 14.7986 | 14.9666 |
| 0.20 | 40.00 | 14.8324 | 14.5602 | 14.2829 | 14.6287 | 14.9666 |
| 0.10 | 100.00 | 12.6491 | 14.5602 | 14.2829 | 14.1067 | 14.9666 |
| 0.10 | 200.00 | 7.7460 | 14.5602 | 14.2829 | 13.1909 | 11.1355 |
|  |  |  |  |  |  |  |
| Expected Utility | 14.6174 | 14.5602 | $\mathbf{1 4 . 6 2 4 7}$ | 14.6187 | 14.5835 |  |

## Optimal Deductible

- Next, we show that the optimal deductible must be nonzero if the premium includes a nonzero proportional loading factor.
- Retrace previous analysis; initial wealth $=W_{0}, \pi=$ loss probability, and premium $P=(1+\lambda) \pi(L-d)$.
- Uninsured lottery $\rightarrow\left(\left\langle W_{0}-L, W_{0}\right\rangle,\langle\pi, 1-\pi\rangle\right)$, insurance lottery $\rightarrow\left(\left\langle W_{0}-P-d, W_{0}-P\right\rangle,\langle\pi, 1-\pi\rangle\right)$.
- Maximand $\rightarrow \max _{d} E(U(W))=\pi U\left(W_{0}-(1+\lambda) \pi(L-d)-d\right)$

$$
\begin{equation*}
+(1-\pi) U\left(W_{0}-(1+\lambda) \pi(L-d)\right) \tag{15}
\end{equation*}
$$

## Optimal Deductible

- FOC: $(1-\pi) \pi(1+\lambda) U^{\prime}\left(W_{0}-(1+\lambda) \pi(L-d)\right)$

$$
\begin{equation*}
=(1-\pi(1+\lambda)) \pi U^{\prime}\left(W_{0}-(1+\lambda) \pi(L-d)-d\right) \tag{16}
\end{equation*}
$$

- Suppose $\lambda=0$. Then

$$
\begin{equation*}
U^{\prime}\left(W_{0}-\pi(L-d)\right)=U^{\prime}\left(W_{0}-\pi(L-d)-d\right) \tag{17}
\end{equation*}
$$

i.e., $d=0$ (Bernoulli Principle).

- Now suppose $\lambda>0$; then $d>0$. If $d$ is not positive, then (16) does not obtain!


## Optimal Deductible

- Suppose $U=\ln W$. Then the first order condition is

$$
\begin{equation*}
\frac{(1-\pi) \pi(1+\lambda)}{W_{0}-\pi(1+\lambda)(L-d)}=\frac{(1-\pi(1+\lambda)) \pi}{W_{0}-\pi(1+\lambda)(L-d)-d} \tag{18}
\end{equation*}
$$

- Solving for $d$, we obtain

$$
\begin{equation*}
d=\frac{\lambda\left(\pi(1+\lambda) L-W_{0}\right)}{(1+\lambda)(\pi(1+\lambda)-1)} . \tag{19}
\end{equation*}
$$

- Since the signs of both the numerator and denominator are negative for $\lambda>0$, this implies $d>0$ with actuarially unfair premiums (also note that $\lambda=0 \Rightarrow d=0$ ).


## Comparative Statics

- $\frac{\partial d}{\partial \lambda}=\frac{W_{0}}{(1+\lambda)^{2}}+\frac{(\pi-1) \pi\left(L-W_{0}\right)}{(\pi(1+\lambda)-1)^{2}}>0$; i.e., the optimal deductible is positively related to the premium loading,
- $\frac{\partial d}{\partial \pi}=\frac{\lambda\left(W_{0}-L\right)}{(\pi(1+\lambda)-1)^{2}}>0$; i.e., the optimal deductible is positively related to loss frequency,
- $\frac{\partial d}{\partial W_{0}}=\frac{\lambda}{(1-\pi(1+\lambda))(1+\lambda)}>0$; i.e., the optimal deductible is positively related to the level of initial wealth, and
- $\frac{\partial d}{\partial L}=\frac{\pi \lambda}{(\pi(1+\lambda)-1)}<0$; i.e., the optimal deductible is inversely related to loss severity.


## Moral Hazard

- "Hidden Action" problem
- Moral hazard is the risk that a party to a contract will subsequently deviate from the terms of the contract.
- Moral hazard is thus a problem created by information asymmetry after the transaction occurs.


## Moral Hazard

In the absence of insurance, expected wealth $(E(W))$ is written as

$$
\begin{equation*}
E(W)=W_{0}-c(s)-p(s) L . \tag{1}
\end{equation*}
$$

Next, we maximize $E(W)$ by differentiating (1) with respect to $s$ and solving for the value of $s^{*}$ that causes the resulting equation to be equal to zero:

$$
\begin{equation*}
\frac{d \mathrm{E}(W)}{d s}=-c^{\prime}\left(s^{*}\right)-p^{\prime}\left(s^{*}\right) L=0 . \tag{2}
\end{equation*}
$$

Rearranging (2), we obtain a very familiar result; the optimal level of safety occurs when the marginal cost of safety $\left(c^{?}\left(s^{*}\right)\right)$ is equal to the marginal benefit of safety $\left(-p^{\prime}\left(s^{*}\right) L\right)$; i.e.,

$$
\begin{equation*}
c_{\text {Marginal Cost }}^{c^{\prime}\left(s^{*}\right)}=\underbrace{-p^{\prime}\left(s^{*}\right) L}_{\text {Marginal Benefit }} . \tag{3}
\end{equation*}
$$

## Moral Hazard

Next, we introduce insurance in which the insurer covers the proportion $\alpha$ of the risk for a premium of $\$ \alpha P$. Thus expected wealth is

$$
\begin{equation*}
E(W)=W_{0}-c(s)-(1-\alpha) p(s) L-\alpha P \tag{4}
\end{equation*}
$$

and $s^{*}$ is determined by the following equation:

$$
\begin{gather*}
\frac{d E(W)}{d s}=-c^{\prime}\left(s^{*}\right)-(1-\alpha) p^{\prime}\left(s^{*}\right) L=0 ; \text { consequently, }  \tag{5}\\
c_{\text {Marginal Cost }}^{\prime}\left(s^{*}\right)=\underbrace{-(1-\alpha) p^{\prime}\left(s^{*}\right) L}_{\text {Marginal Benefit }} \tag{6}
\end{gather*}
$$

Since coinsurance proportionately scales down the marginal benefit of safety, $s^{*}$ is lower when insurance is purchased. If $\alpha=1$, then there is no benefit to investing in safety; consequently, the optimal value for $s^{*}$ is $s^{*}=0$. Herein lies the moral hazard problem.

## Moral Hazard

Marginal benefit

Marginal benefit with<br>- - - - - - - no hedge<br>$-\cdots-\cdots-\cdots-50 \%$ hedge<br>- - - $\mathbf{1 0 0 \%}$ hedge



## Moral Hazard

The solution to this dilemma is for the insurer to make the premium itself a function of the level of investment in safety. In other words, let $P=P(s)$, where $P^{\prime}(s)<0$. Thus (4) is rewritten as:

$$
\begin{equation*}
E(W)=W_{0}-c(s)-(1-\alpha) p(s) L-\alpha P(s), \tag{7}
\end{equation*}
$$

and $s^{*}$ is determined by the following equation:

$$
\begin{equation*}
\frac{d \mathrm{E}(W)}{d s}=-c^{\prime}\left(s^{*}\right)-(1-\alpha) p^{\prime}\left(s^{*}\right) L-P^{\prime}\left(s^{*}\right)=0 . \tag{8}
\end{equation*}
$$

Thus the equilibrium condition of marginal cost equaling marginal benefit is written as follows:

$$
\begin{equation*}
c_{\text {Marginal Cost }}^{c^{\prime}\left(s^{*}\right)}=\underbrace{-(1-\alpha) p^{\prime}\left(s^{*}\right) L-\alpha P^{\prime}(s)}_{\text {Marginal Benefit }} . \tag{9}
\end{equation*}
$$

In (9), even when $\alpha=1$, it will be optimal to invest in safety, since the premium charged is sensitive to the level of investment in safety.

## Moral Hazard



## Moral Hazard

- Risk transfer creates moral hazard
- Contractual and pricing strategies for mitigating moral hazard:
- Risk sharing (partial insurance)
- Experience rating


## Adverse Selection

## - "Hidden Information" problem

- Adverse selection is the risk that the party who wants to enter into a contract agreement with you is most likely to produce an undesirable outcome.
- Adverse selection is the problem created by information asymmetry before the transaction occurs.


## Adverse Selection

- Examples of adverse selection
- Insurers know less about the true risk characteristics of their policyholders than the policyholders themselves.
- When a firm hires a worker, it knows less than the worker about his abilities.
- The seller of a used car has more information about the car than the potential buyers.


## Adverse Selection in Insurance

- Two states of the world (loss and no loss), and two driver types (high accident probability ( $p_{H}=75 \%$ ) and low accident probability $\left(p_{L}=25 \%\right)$ ).
- Otherwise, drivers are identical in all respects; $W_{0}=\$ 125$ and $L=\$ 100$. Thus if there are no transactions costs, $E\left(W_{L}\right)=W_{0}-E\left(L_{L}\right)=\$ 125-.25(\$ 100)=\$ 100$ for low risk drivers, and

$$
\begin{gathered}
E\left(W_{H}\right)=W_{0}-E\left(L_{H}\right)=\$ 125-.75(\$ 100)=\$ 50 \text { for high risk } \\
\text { drivers. }
\end{gathered}
$$

- With premiums set at the expected value of loss for each insured ( $\$ 25$ for low risk drivers and $\$ 75$ for high risk drivers), the Bernoulli principle implies that each would fully insure.


## Adverse Selection in Insurance

- Suppose that we cannot identify the low and high risk drivers. However, we do know that there are equal numbers of low and high risk drivers. What to do?
- One possibility - Charge an average premium of $\$ 50$. What's wrong with this strategy?
- High risk drivers now receive even more utility from insuring. However, low risk drivers cancel their policies because the expected utility of being uninsured is higher than the expected utility of being insured.
- Consequently, the insurer is stuck with a portfolio of high-risk drivers and an inadequate premium.


## Adverse Selection in Insurance



## Adverse Selection Limits Insurability!

Figure 4. Rothschild Stiglitz Separating Equilibrium


## Practical Implications of Rothschild-Stiglitz

- The Rothschild-Stiglitz "separating equilibrium" model shows that an insurer can mitigate adverse selection by limiting the set of contract choices offered to consumers.
- In the "real world", insurers anticipate that bad risks will select lower deductibles than good risks; consequently, insurers adjust low deductible insurance policy premiums to reflect the anticipated cost of adverse selection.
- Therefore, one who is a good risk ought to select high deductible insurance policies!

