# THEORY OF RISK CAPITAL IN FINANCIAL FIRMS

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This paper develops a concept of risk capital that can be applied to the financing, capital budgeting, and risk management decisions of financial firms. The development focuses particularly on firms that act as a *principal* in the ordinary course of business. Principal activities can be asset-related, as in the case of lending and block-positioning; liability-related, as in deposit-taking and writing of guarantees (including insurance, letters of credit, and other contingent commitments); or both, as in the writing of swaps and other derivatives for customers.

For the purposes of this paper, principal financial firms have three important distinguishing features. The first is that their customers can be major liabilityholders; for example, policyholders, depositors, and swap counterparties are all liabilityholders as well as customers. By definition, a financial firm's customers strictly prefer to have the payoffs on their contracts as unaffected as possible by the fortunes of the issuing firm. Hence, they strongly prefer firms of high credit quality. Investors, by contrast, expect their returns to be affected by the profits and losses of the firm. Hence, they are less credit-sensitive provided, of course, they are compensated appropriately for risk. This means that A-rated firms, for example, can generally raise the funds they need to operate, but are at a disadvantage in competing with AAA-rated firms in businesses such as underwriting insurance or issuing swaps. The presence of credit-sensitive customers thus greatly increases the importance of risk control of the overall balance sheet.[1]

A second distinguishing feature of principal firms is their *opaqueness* to customers and investors.[2] That is, the detailed asset holdings and business activities of the firm are not publicly disclosed (or, if disclosed, only with a considerable lag in time). Furthermore, principal financial firms typically have relatively liquid balance sheets that, in the course of only weeks, can and often do undergo a substantial change in size and risk.[3] Unlike manufacturing firms, principal financial firms can enter, exit, expand, or contract individual businesses quickly at relatively low cost. These are changes that customers and investors cannot easily monitor. Moreover, financial businesses—even non-principal businesses like mutual-fund management—are susceptible to potentially enormous "event risk" in areas not easily predictable or understood by outsiders.[4]

All of this implies that principal firms will generally experience high "agency" and "information" costs in raising equity capital and in executing various types of customer transactions.[5] (We later refer to these "dissipative" or "deadweight" costs collectively as *economic costs of risk capital*, in a manner to be made more precise.) Risk management by the firm is an important element in controlling these costs.

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1. For an elaboration on the difference between "customers" and "investors" of the financial-service firm as a core concept, see Robert C. Merton, (1992), *Continuous-Time Finance*, Revised Edition, Oxford: Basil Blackwell; R.C.Merton (1993), "Operation and Regulation in Financial Intermediation: A Functional Perspective," in Peter Englund, ed., *Operation and Regulation of Financial Markets*, Stockholm: The Economic Council: 17-67; and R.C. Merton and Zvi Bodie, "On the Management of Financial Guarantees," *Financial Management*, 21 (Winter, 1992): 87-109.

2. The notion of "opaqueness" of financial institutions is developed by Stephen Ross in "Institutional Markets, Financial Marketing, and Financial Innovation," *Journal of Finance*, 44 (July, 1989): 541-556. For further discussion, see Merton (1993), cited in note 1.

3. As reported in *The Wall Street Journal*, October 24, 1991, the investment bank of Salomon Brothers reduced its total assets or "footings" by $50 billion in a period of approximately 40 days.

4. For example, consider the potentially large exposures from the "scandals" at E.F. Hutton (check writing), Merrill Lynch ("ticket in drawer"), Salomon Brothers (Treasury auction), Drexel Burnham Lambert (FIRREA/collapse of high-yield debt market), and T. Rowe Price Associates (money-market-fund credit loss).

5. For detailed development and review of the literature on asymmetric information and agency theory in a financial market context, see Amir Barnea, Robert Haugen, and Lemma Senbet (1985), *Agency Problems and Financial Contracting*, Englewood Cliffs, NJ: Prentice Hall; Michael Jensen, (1986), "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," *American Economic Review*, 76 (May): 323-329. and especially N. Strong and M. Walker (1987), *Information and Capital Markets*, Oxford: Basil Blackwell.

A third distinguishing feature of principal financial firms is that they operate in competitive financial markets. Their profitability is thus highly sensitive to their cost of capital, and especially their cost of risk capital. Allocating the costs of risk capital to individual businesses or projects is a problem for organizations that operate in a more or less decentralized fashion. As we shall discuss, there is no simple way to do so. Moreover, any allocation must necessarily be *imputed*, if only because highly risky principal transactions often require little or no up-front expenditure of cash.

For example, an underwriting commitment can be executed with no immediate cash expenditure. However, the customer counterparty would not enter into the agreement if it did not believe that the underwriting commitment would be met. The commitment made by the underwriting business is backed by the entire firm. Therefore, the strength of this guarantee is measured by the overall credit standing of the firm. The problem of capital allocation within the firm is thus effectively the problem of correctly charging for the guarantees provided by the firm to its constituent businesses.

These three distinctive features of principal financial firms—credit-sensitivity of customers, high costs of risk capital (resulting from their opaqueness), and high sensitivity of profitability to the cost of risk capital—should all be taken into account explicitly by such firms when deciding which activities to enter (or exit), how to finance those activities, and whether to hedge its various market or price exposures.

**What is Risk Capital?** We define *risk capital* as *the smallest amount that can be invested to insure the value of the firm's net assets against a loss in value relative to the risk-free investment of those net assets.* By *net assets*, we mean gross assets minus customer liabilities (valued as if these liabilities are default-free). Customer liabilities can be simple fixed liabilities such as guaranteed insurance contracts (GICs), or complex contingent liabilities such as property and casualty insurance policies. With fixed customer liabilities, the riskiness of net assets (as measured, for example, by the standard deviation of their change in value) is the same as the riskiness of gross assets. With contingent customer liabilities, however, the riskiness of net assets depends not only on the riskiness of gross assets, but also on the riskiness of customer liabilities and the covariance between changes in the value of gross assets and changes in the value of customer liabilities. The volatility of the change in the value of net assets is the most important determinant of the amount of risk capital.

As defined, risk capital differs from both *regulatory capital*, which attempts to measure risk capital according to a particular accounting standard, and from *cash capital*, which represents the up-front cash required to execute a transaction. Cash capital is a component of *working capital* that includes financing of operating expenses like salaries and rent. Cash capital can be large, as with the purchase of physical securities—or small, as with futures contracts and repurchase agreements—or even negative, as with the writing of insurance.

The organization of the paper is as follows. In the next section, a series of examples is presented to show that the amount of risk capital depends only on the riskiness of net assets, and not at all on the form of financing of the net assets. These examples further establish how risk capital funds, provided mainly by the firm's shareholders (except in the case of extremely highly leveraged firms), are then either implicitly or explicitly used to purchase asset insurance from various sources. Besides third-party guarantors, other potential issuers of asset insurance to the firm are the firm's stakeholders, including customers, debtholders, and shareholders.

We next discuss how standard methods of accounting can fail to measure risk capital and its associated costs correctly in the calculation of firm profitability, and how this can lead to an overstatement of profitability. The economic costs of risk capital to the firm are shown to be the "spreads" on the price of asset insurance arising from information costs (adverse selection and moral hazard) and agency costs. We then use this framework to establish the implications for hedging and risk-management decisions.

Finally, for multi-business firms, we discuss the problems that arise in trying to allocate the risk capital of the firm among its individual businesses. It is shown that, for a given configuration, the risk capital of a multi-business firm is less than the aggregate risk capital of the businesses on a stand-alone basis. Therefore, full allocation of risk capital across the individual businesses of the firm is generally not feasible, and attempts at such a full allocation can significantly distort the true profitability of individual businesses.

## MEASURING RISK CAPITAL

We now use a series of hypothetical but concrete examples to illustrate the concept of risk capital. In the first set of examples, there are no customer liabilities, so that gross assets equal net assets. After that, we consider two cases with customer liabilities, one with fixed liabilities and the other with contingent liabilities.

Consider the hypothetical newly-formed firm of Merchant Bank, Inc., a wholly owned subsidiary of a large AAAA-rated[6] conglomerate. The firm currently has no assets. Merchant Bank's one and only deal this year will be a $100 million participation in a one-year bridge loan promising 20% interest ($120 million total payment at maturity). It does not plan to issue any customer liabilities. Merchant Bank's net assets will thus consist of this single bridge loan.

The bridge loan is a risky asset. We assume in particular that there are only three possible scenarios: A likely "anticipated" scenario, in which the loan pays off in full the promised $120 million; an unlikely "disaster" scenario, in which the borrower defaults but at maturity the lender recovers 50 cents on the dollar—that is, collects $60 million; and a rare "catastrophe" scenario, in which the lender recovers nothing.

To invest in the bridge loan requires $100 million of *cash* capital. Because this asset is risky, the firm also needs risk capital.

Merchant Bank wants to finance the cash capital by means of a one-year note issued to an outside investor. The firm wants the note to be default free. If these terms can be arranged, then at the current riskless rate of 10%, $110 million would be owed the noteholder at maturity.

In general, a firm has essentially two ways to eliminate the default risk of its debt liabilities. Both involve the purchase of insurance: The first is to do so indirectly through the purchase of insurance on its *assets*; the second and more direct method is to purchase insurance on its (debt) *liabilities*. (Combinations of these would also work.) As we shall see, the two are economically equivalent. The risk capital of the firm is equal to the smallest investment that can be made to obtain complete default-free financing of its net assets.

**Risk Capital and Asset Guarantees.** Suppose that Merchant Bank buys insurance on the bridge loan from a AAAA-rated bond insurer. Suppose further that, for $5 million, Merchant Bank can obtain insurance just sufficient to guarantee a return of $110 million on the bridge loan.[7] With this asset insurance in place, the value of Merchant Bank's assets at the end of the year will equal or exceed $110 million. The noteholders of Merchant Bank are thus assured of receiving the full payment of their interest and principal, and the note will be default-free.

It follows from the definition of risk capital that the price of the loan insurance ($5 million) is precisely the amount of risk capital Merchant Bank requires if it holds the bridge loan. Merchant Bank would need to fund it with a $5 million cash equity investment from its parent. Once these transactions have been completed, Merchant Bank's accounting balance sheet will be as follows:

**ACCOUNTING BALANCE SHEET A**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (default free) | $100 |
| Loan insurance (from insurance company) | 5 | Shareholder equity | 5 |

If the bridge loan pays off as promised at the end of the year, Merchant Bank will be able to return a total of $10 million pre-tax to its parent ($20 million in interest income less $10 million in interest expense). If the bridge loan defaults, the asset insurance covers any shortfall up to $110 million, and Merchant Bank will just be able to meet its note obligations. There will be nothing to return to the parent. The risk capital used to purchase the insurance will have been just sufficient to protect the firm from any loss on the underlying asset (including financing expense of the cash capital). And, of course, the risk capital itself will have been lost. In this arrangement, the insurance company bears the risk of the asset; Merchant Bank's parent as shareholder bears the risk of loss of the risk capital itself.

The payoffs (cash flows) at maturity to the various stakeholders in Merchant Bank can be summarized in the following table:

6. By "AAAA-rated", we mean a firm with default-free liabilities that without question will stay that way.

7. That is, *full insurance*. The insurance would take the form of paying Merchant Bank the difference between the promised debt payments and actually received cash flows on the bridge loan.

**Risk capital differs from both *regulatory capital*, which attempts to measure risk capital according to a particular accounting standard, and from *cash capital*, which represents the up-front cash required to execute a transaction.**

**TABLE A: PAYOFF STRUCTURE**

| Bridge Loan | Loan Insurance | Bridge Loan + Insurance | Firm Stakeholders Note | Shareholder |
|---|---|---|---|---|
| ANTICIPATED SCENARIO | | | | |
| 120 | 0 | 120 | 110 | 10 |
| DISASTER SCENARIO | | | | |
| 60 | 50 | 110 | 110 | 0 |
| CATASTROPHE SCENARIO | | | | |
| 0 | 110 | 110 | 110 | 0 |

Note that, in this example, Merchant Bank's accounting balance sheet corresponds to what we shall call the firm's *risk-capital balance sheet*:

**RISK-CAPITAL BALANCE SHEET A**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (default free) | $100 |
| Loan insurance (from insurance company) | 5 | Risk capital | 5 |

By inspection of the two balance sheets, "shareholder equity" is equal to the firm's risk capital, and the non-equity liabilities are default free. We shall see, however, that the accounting and risk-capital balance sheets are in general quite different.

**Risk Capital and Liability Guarantees**. A parent guarantee of the note is an alternative, and perhaps the most common, form of credit enhancement for the debt of a subsidiary such as Merchant Bank.[8] This way, the parent makes no cash equity investment in Merchant Bank. At the outset, the firm's accounting balance sheet is as follows:

**ACCOUNTING BALANCE SHEET B**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (default free) | $100 |
| | | Shareholder equity | 0 |

Here Merchant Bank again obtains the necessary $100 million in cash capital through issuance of a default-free note; however, all asset risk is now borne by the parent. Thus the risk capital is merely taking the form of the parent guarantee of the note. This guarantee is an additional asset of the subsidiary—one that does not appear on its balance sheet. Suppose that the value of this guarantee is worth $G million. Then the parent's (off-balance-sheet) equity investment in Merchant Bank is worth $G million, and Merchant Bank's balance sheet can be restated in terms of its risk-capital balance sheet as follows:[9]

**RISK-CAPITAL BALANCE SHEET B**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (default free) | $100 |
| Note guarantee (from parent) | G | Risk capital | G |

As in the previous example, if the bridge loan pays off as promised, Merchant Bank will be able to return a total of $10 million pre-tax to its parent ($20 million in interest income less $10 million in interest expense). If the bridge loan defaults, so too will Merchant Bank on its note, and the noteholder either collects any unpaid amounts from the parent, or the parent pays out the promised $110 million and receives back the value of the bridge-loan asset seized; either way the economic effect is the same. Merchant Bank of course will have nothing to return to its parent as equityholder. In this arrangement, the parent bears the risk of the asset as guarantor of its subsidiary's debt; the parent also bears the risk of loss of the risk capital as shareholder of Merchant Bank. Table B summarizes in terms of payoffs at maturity:

**TABLE B: PAYOFF STRUCTURE**

| Bridge Loan | Note sans Guarantee | Note Guarantee | Note + Guarantee | Shareholder |
|---|---|---|---|---|
| ANTICIPATED SCENARIO | | | | |
| 120 | 110 | 0 | 110 | 10 |
| DISASTER SCENARIO | | | | |
| 60 | 60 | 50 | 110 | 0 |
| CATASTROPHE SCENARIO | | | | |
| 0 | 0 | 110 | 110 | 0 |

A comparison of Table A and Table B demonstrates the economic equivalence of liability insurance and asset insurance.[10] In both, the noteholder

8. This insurance could take the form of the parent either paying the noteholder the $110 million promised payment in the event of default, and then seizing Merchant Bank's assets, or paying the noteholder the difference between the promised payment and actual payments Merchant Bank is able to make. The parent guarantee avoids outside lenders becoming involved in any bankruptcy of the subsidiary, and gives the parent some "choice." For our purposes here, we can abstract from such details of structure.

9. For a real-world application of this "extended" balance-sheet approach to capture the "hidden" asset and corresponding equity investment arising from parent guarantees of its subsidiary's debt, see R.C. Merton, (1983), "Prepared Direct Testimony of Robert C. Merton on Behalf of ARCO Pipe Line Company," Federal Energy Regulatory Commission, Washington, D.C., Docket No. OR78-1-011 (Phase II), Exhibits II N-C-34-0-34-4 (November 28). For a similar approach to analyze corporate pension assets and liabilities and the firm's guarantee of any shortfall on the pension plan, see Zvi Bodie (1990), "The ABO, the PBO, and Pension Investment Policy," *Financial Analysts Journal*, 46 (September/October): 27-34.

10. This equivalence may not apply exactly if one takes account of the various bankruptcy costs and delays in payments which could occur, for example, if Merchant Bank sought Chapter 11 bankruptcy protection.

bears no risk and the parent, solely in its capacity as shareholder of Merchant Bank, obtains the same cash flows: $10 million in the "anticipated" scenario and zero otherwise. Moreover, the note guarantee has the same cash flows as the bridge-loan insurance. The note guarantee therefore is also worth G = $5 million. Thus, risk capital is once again $5 million.[11]

**Liabilities with Default Risk**. We now turn to the more typical case where our hypothetical firm, Merchant Bank, is willing to issue liabilities with some default risk. Suppose it issues the same 10% note (promising $110 million at maturity), but without any of the credit enhancements of the previous case. This now risky note will sell at a discount $D to par (at a promised yield to maturity higher than 10%), leaving Merchant Bank $D short of its need for $100 million cash capital. The shortfall in initial funding must be supplied in the form of a cash equity investment. Merchant Bank's beginning balance sheet is as follows:

**ACCOUNTING BALANCE SHEET C**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (risky) | $100 – D |
| | | Shareholder equity | D |

Once again, if the bridge loan pays off as promised, Merchant Bank will be able to pay a total of $10 million pre-tax to its parent.[12] If the bridge loan defaults, so too will Merchant Bank default on its note, and the noteholder will be at risk for any shortfall on the bridge loan under $110 million. Merchant Bank will have nothing to return to its parent.

Merchant Bank's shareholder here receives the same payoffs as it did in the previous examples (see Table C). This economic equivalence implies that the firm's equity must be worth D = $5 million initially. Correspondingly, the risky note will have an initial value of $95 million (with a *promised* yield to maturity of $15 on $95, or 15.8%).

To see where risk capital enters, consider the position of the debtholder. The debtholder can interpret its purchase of the risky note as equivalent to the following three-step transaction: First, the purchase of default-free debt from Merchant Bank for $100 million; second, the sale to Merchant Bank of debt insurance for $5 million; and third, the netting of payments owed the debtholder on the default-free debt against payments owed the firm if the insurance is triggered. It is perhaps easiest to see this by observing the economic identity:[13]

Risky note + note insurance = Default-free note
so that
Risky note = Default-free note – note insurance.

As already shown (in Tables A and B), note insurance is economically equivalent to asset insurance. Thus, the debtholder can interpret its purchase of the risky note as equivalent to the purchase of default-free debt coupled with the *sale* to Merchant Bank of *asset* insurance (on the bridge loan) for $5 million. In other words:

Risky note = Default-free note – asset insurance.

This relation allows the restatement of the accounting balance sheet C in its risk-capital form:

**RISK-CAPITAL BALANCE SHEET C**

| | | | |
|---|---|---|---|
| Bridge loan | $100 | Note (default free) | $100 |
| Asset insurance (from note holder) | 5 | Risk capital | 5 |

The payoffs at maturity associated with this risk-capital balance sheet are shown in Table C:

**TABLE C: PAYOFF STRUCTURE**

| Bridge Loan | Asset Insurance | Default-free Note | Risky Note = Default-free Note – Asset Insurance | Shareholder |
|---|---|---|---|---|
| ANTICIPATED SCENARIO | | | | |
| 120 | 0 | 110 | 110 | 10 |
| DISASTER SCENARIO | | | | |
| 60 | 50 | 110 | 60 | 0 |
| CATASTROPHE SCENARIO | | | | |
| 0 | 110 | 110 | 0 | 0 |

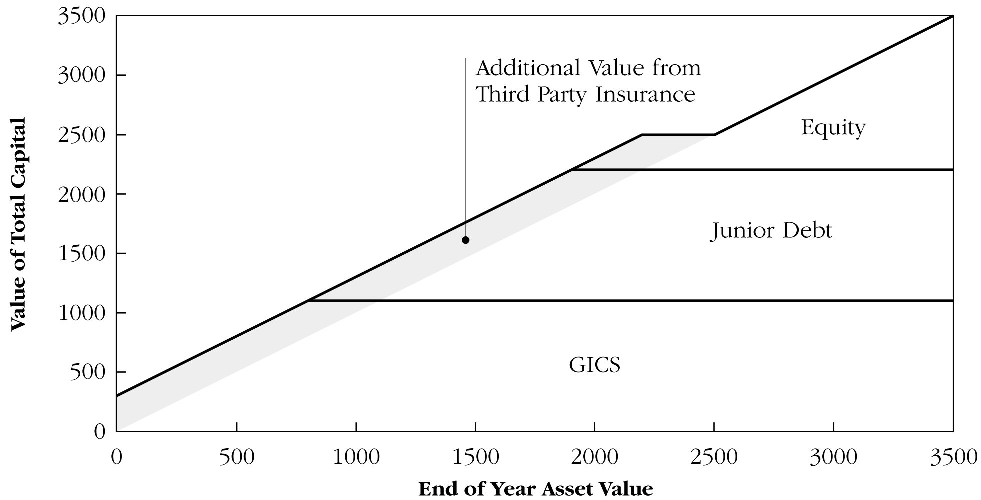
11. The assumption that economically-equivalent cash flows have the same value is made only for expositional convenience in this part of the paper. Later in the discussion of the management of risk capital, the assumption is relaxed to take account of differences in information and agency costs among alternative guarantors.

12. $20 million in interest income less $15 million in cash plus amortized interest expense plus $5 million return of capital.

13. For a full development and applications of this identity, see R.C. Merton, (1990), "The Financial System and Economic Performance," *Journal of Financial Services Research*, 4 (December): 263-300; and Merton and Bodie (1992), cited in note 1.

**Each of the examples has a different accounting balance sheet. Yet all have very similar risk-capital balance sheets. They have the same amount of risk capital—because the underlying asset requiring the risk capital is the same in all cases.**

**FIGURE 1**
PAYOFFS TO FIRM CAPITAL PROVIDERS




Each of the examples (A,B,C) has a different accounting balance sheet. Yet all have very similar risk-capital balance sheets. They have the same amount of risk capital—because the underlying asset requiring the risk capital is the same in all cases. They differ only in which parties bear the risk of insuring the asset: the insurance company (example A), the parent (example B), or the noteholder (example C).

**A More General Case.** The concept of risk capital is now further expanded by analyzing a more general balance sheet. The goals here are to illustrate the case of fixed customer liabilities and the purchase of asset insurance from multiple sources.

Consider a firm with an investment portfolio of risky assets worth $2.5 billion. The firm has customer liabilities outstanding in the form of one-year guaranteed investment contracts (GICs) promising 10% on their face value of $1 billion. Because the riskless rate is also 10%, the *default-free* value of these customer liabilities is $1 billion. The net assets—equal to assets minus the default-free value of customer liabilities—are thus worth $1.5 billion.

The riskiness of the portfolio is assumed to be such that the price of insurance to permit the portfolio to be financed risklessly for a year is $500 million. Since the customer liabilities are fixed, it follows that the price of insurance to permit the *net* assets to be financed risklessly for a year is also $500 million.[14] Therefore, $500 million is the required risk capital based on a one-year horizon.

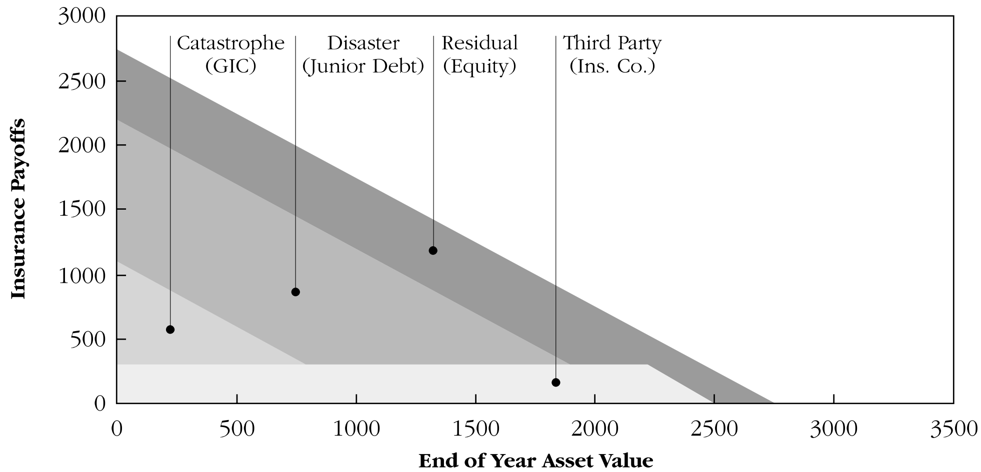
The firm's investor financings are in two forms: one-year junior debt promising 10% on its face value of $1 billion and shareholder equity. Thus, the total promised payment on fixed liabilities at the end of the year is $2.2 billion, comprised of $1.1 billion of GICs and $1.1 billion of debt that is junior to the GICs.

Suppose that the firm has formally obtained *partial* insurance on its investment portfolio, arbitrarily chosen to cover the *first* $300 million of decline of value of portfolio value below $2.5 billion. The insurance is thus structured to guarantee the portfolio value at $2.5 billion at year end, but is capped at a maximum payout of $300 million; therefore, the cap will be reached if the portfolio value falls below $2.2 billion. Assume, moreover, that the value of this "third-party" insurance is $200 million. The value of the policy appears as an additional asset on the firm's accounting balance sheet.

Figure 1 shows the payoffs on the various liabilities of the firm depending on the value of the investment portfolio at year end. Because the portfolio is only partially protected from loss by the firm-owned insurance policy, the junior debt and the customer liabilities are both potentially at risk to receive less than their promised payments.

14. By the end of the year, the *gross* assets will have experienced a loss relative to a risk-free investment if they fall below $2,750 million (110% of $2,500 million). The *net* assets will have experienced a loss relative to a risk-free investment if they fall below $1,650 million (110% of $1,500 million). Since year-end net assets always equals year-end gross assets minus $1,100 million, any shortfall in year-end *gross* assets is exactly equal to the shortfall in year-end *net* assets, and vice versa. Therefore, the loss to the insurer of gross assets is identical to the loss to the insurer of net assets, and the prices of the two policies are the same.

**FIGURE 2**
COMPONENTS OF ASSET INSURANCE




As the senior liability, the GICs are most protected against a decline in the firm's asset values. As shown in Figure 1, customers holding the GICs are at risk only if the value of the firm's portfolio has fallen below $800 million at year end, a decline in value of more than 68%. Accordingly, the GICs trade at only a small percentage discount to par. In our example, we assume that this discount is 1%, thus implying a price of $990 million and a promised yield to maturity of 11% ($110 on $990).

The junior debt is considerably riskier: the holders are exposed to loss if the value of the firm's portfolio falls below $1.9 billion by year end, a decline of about 24%. This debt therefore will trade at a larger discount to par. In our example, we assume that the discount is 10% for a price of $900 million, with a promised yield to maturity of 22.2% ($200 on $900). The value of the firm's equity is equal to $810 million, the difference between the value of total assets ($2.7B) and the market value of customer- and investor-held liabilities ($990 + 900MM). The accounting balance sheet (valuing assets and liabilities at market) is thus as follows:

**ACCOUNTING BALANCE SHEET D**

| | | | |
|---|---|---|---|
| Investment portfolio | $2,500 | GICs (par $1,000) | $990 |
| "Third-party" insurance | 200 | Debt (par $1,000) | 900 |
| (insurance company) | | Equity | 810 |
| Total assets | 2,700 | Total liabilities | 2,700 |

We now construct the risk-capital balance sheet for this firm. As in our earlier discussion of liabilities with default risk, the economic interpretation of the GIC holders is that, in effect, they have purchased default-free GICs and simultaneously *sold* some asset insurance to the firm, with the two transactions netted against each other. GIC holders are at risk only in the least likely of circumstances, and so they provide a kind of "catastrophe" insurance. As shown in Figures 1 and 2, the catastrophe insurance pays off only if the portfolio value falls by more than 68%. The (implicit) price of this insurance is the discount from the default-free value of the GICs, or $10 million ($1 billion – $990 million). Similarly, the debtholders' position is as if they purchased default-free debt and simultaneously sold to the firm asset insurance with a value of $100 million ($1 billion – 900 million). This insurance pays off if the firm's portfolio falls below $1.9 billion, but the maximum payoff is capped at $1.1 billion. The risk to the debtholders is greater than the risk to the GIC holders, but is still relatively small. As illustrated in Figure 2, it is a kind of "disaster" insurance.

We have so far accounted for total premiums of $310 million for asset insurance (third-party ($200mm) + debtholders ($100mm) + GICs ($10mm)). But we know that it takes $500 million in premiums to insure the portfolio fully. Hence, the balance of the insurance representing $190 million in premiums must effectively be provided by the equityholders. Because this insurance covers all the risks not covered by the other kinds of insurance, we call it "residual" insurance. (Figure 2 shows the combination of all sources of asset insurance.)

The total insurance has the same payoff structure as a *put option* on the portfolio with an exercise

**Customers and other senior providers of cash capital are typically sellers of catastrophe type insurance—the kind that is called upon to pay in only the rarest of instances. "Mezzanine" debtholders and equityholders provide cash capital and sell to the firm almost all the insurance not purchased from third-party providers.**

price equal to the current value of the portfolio ($2.5 billion) plus one year of interest at the riskless rate ($250 million), or $2.75 billion. The aggregate value of this asset insurance, or "put option," is equal to $500 million—as assumed at the outset. This is the risk capital of the firm.[15]

The equityholders can think of their $810 million investment as serving three functions: providing $500 million of default-free cash-capital financing (bringing the total cash capital to $2.5 billion), providing $500 million of risk capital to pay for asset insurance, and selling to the firm a portion of that asset insurance worth $190 million. The equityholders' net cash contribution is $500 plus $500 minus $190 million, which equals $810 million.

The risk-capital balance sheet of the firm is as follows:

**RISK-CAPITAL BALANCE SHEET D**

| | | | |
|---|---|---|---|
| Asset portfolio | $2,500 | Cash capital (default free) | |
| | | Customers (GICs) | $1,000 |
| Asset insurance | | Debtholders | 1,000 |
| Equityholders ("residual") | 190 | Equityholders | 500 |
| Insurance Co ("third-party") | 200 | Total cash capital | 2,500 |
| Debtholders ("disaster") | 100 | | |
| Customers ("catastrophe") | 10 | Risk capital (Equityholders) | 500 |
| Total insurance | 500 | | |
| Total assets | 3,000 | Total Capital | 3,000 |

This balance sheet encapsulates three basic functions of capital providers. First, *all provide cash capital.* Second, *all are sellers of asset insurance* to the firm, although in varying degrees. Customers and other senior providers of cash capital are typically sellers of catastrophe type insurance—the kind that is called upon to pay in only the rarest of instances. This level of exposure is typical because customers prefer to have their contract payoffs insensitive to the fortunes of the issuing firm. Customers will buy contracts from the firm only if they perceive the risk of default on those contracts to be very low. "Mezzanine" debtholders and equityholders are investors who provide cash capital and sell to the firm almost all the insurance not purchased from third-party providers.

The third function is the provision of risk capital, which is the cash required for the purchase of asset insurance. It is almost always performed by equityholders, as in all our illustrations. (Non-equity liabilityholders and other stakeholders in the firm will also be providers of risk capital if the market value of the underlying assets is less than the value of promised liabilities, capitalized at the riskless rate.)

A comparison of the risk-capital balance sheet with the accounting balance sheet thus illustrates that the debt and equity values of the firm need not, and generally will not, sum to the firm's cash capital; nor does the value of the equity necessarily equal the firm's risk capital. Cash capital is determined by the *assets* of the firm. Risk capital is determined by the riskiness of the *net assets* of the firm. Debt and equity, defined in the *institutional* sense, represent the netting of asset insurance against the provision of riskless cash capital and risk capital.

**Contingent Customer Liabilities.** As mentioned earlier, with contingent customer liabilities, the riskiness of net assets will in general differ from the riskiness of gross assets. The following example illustrates this difference.[16]

Consider again a principal financial firm with no equity, but with liabilities fully guaranteed by a AAAA parent. Suppose the firm issues a contingent liability in the form of a one-year S&P 500 index-linked note that promises to pay $100 million times the total return per dollar on the S&P index over the year. The purchaser of the note is a customer, say, a pension fund, that wants the return on its $100 million portfolio to match exactly that of the S&P 500 stock index. The customer has chosen this method

15. An alternative interpretation of the coverage provided by the four sources of insurance is as follows: The equityholders fully insure the gross assets at a level of $2.75 billion by year end, but purchase reinsurance from the insurance company that insures the assets to a level of $2.5 billion by year end. The insurance company in turn purchases reinsurance from the debtholders that insures the gross assets to a level of $1.9 billion by year end. The debtholders then purchase reinsurance from the firm's GIC customers that insures the gross assets to a level of $1.1 billion by year end. Equivalently, this can be expressed in terms of *put options*: The equityholders sell to the firm, for $500 million, a put option on the gross assets with exercise price $2.75 billion. They in turn spend $310 million of the $500 million proceeds to buy a put option from the insurance company with exercise price $2.5 billion. The insurance company then spends $110 million to buy a put option from the debtholders with exercise price $1.9 billion. Finally, the debtholders spend $10 million to purchase a put option from the GIC customers with exercise price $1.1 billion. The equityholders, insurance company, and debtholders have thus each sold a put option on the gross assets at one exercise price, and purchased reinsurance in the form of a second put option at a lower exercise price. For the formal development of the correspondence between loan guarantees and put options, see R.C. Merton (1977), "An Analytical Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory," *Journal of Banking and Finance*, 1 (June): 3-11; see also R. C. Merton (1992,1993), cited in note 1.

16. For an illustration of this point in the case of gross and net assets of a corporate pension plan, see Bodie (1990), cited in note 9.

of investing as an alternative to investing in an S&P 500 index fund. At the instant the transaction is consummated, the firm's accounting balance sheet is as follows:

**ACCOUNTING BALANCE SHEET E**

| | | | |
|---|---|---|---|
| Cash | $100 | Index-linked Note | $100 |
| | | Shareholder equity | 0 |

How the firm chooses to invest the $100 million will determine its risk capital. For instance, the firm might invest in one-year U.S. Treasury bills paying 10%. If it does so, the *gross* assets are riskless, but the *net* assets are extremely risky. In fact, the net assets are equivalent to a short position in the S&P 500.[17] By year end, the parent as guarantor will have to make up a shortfall that is equal to the total return on $100 million worth of the S&P 500 minus $10 million, the return on U.S. Treasury bills, if this amount is positive. This shortfall payment is the same payoff as that promised by a European call option on $100 million worth of the S&P 500 with a strike price of $110 million.[18,19] The risk capital of the firm—the smallest amount that can be invested to insure the value of its net assets—is thus equal to the value of this call option.

As an alternative to U.S. Treasury bills, the firm might invest in the actual portfolio of stocks comprising the S&P 500. Assume it can do so costlessly. In this case, the *gross* assets are risky, but they exactly match the liabilities, so that the *net* assets are *riskless*. When the assets are invested this way, the firm's risk capital is zero.

As another alternative, the firm might invest in a customized portfolio of stocks that tracks fairly closely the S&P 500, but that omits the companies that the firm believes will underperform the S&P 500 index. In this case, the riskiness of the net assets is determined by the potential deviations in performance between the customized portfolio and the index. The risk capital of the firm will equal the value of a guarantee that pays the amount by which the customized portfolio underperforms the index, if it does so at all.[20]

These examples illustrate how the riskiness of the net assets can be significantly less than or greater than the riskiness of the gross assets. They also show that it is the riskiness of *net* assets that determines the type of insurance required to permit default-free financing for the firm, and hence it is the riskiness of net assets that determines the amount of the firm's risk capital.

## ACCOUNTING FOR RISK CAPITAL IN THE CALCULATION OF PROFITS

As discussed above, risk capital is implicitly or explicitly used to purchase insurance on the net assets of the firm from a variety of potential providers. Insurance is a financial asset, and the gains or losses on this asset should be included along with the gains or losses on all other assets in the calculation of profitability. Standard methods of accounting often fail to do this, however. For example, as discussed earlier, when a parent guarantees the performance of a subsidiary, the guarantee is not usually accounted for as an asset on the balance sheet of the subsidiary.

To illustrate, consider a securities underwriting subsidiary of a principal financial firm. The subsidiary anticipates deriving $50 million in revenues from underwriting spreads over the next year. It anticipates customary expenses of $30 million, so that its profit before tax is anticipated to be $20 million. (This profit figure assumes no mishaps such as occurred, for example, in the underwriting of British Petroleum shares in 1986.)[21] The subsidiary has an ongoing net working-capital requirement of $10 million. It has no other formal assets or liabilities and so its equity capital is $10 million.

Thus, the subsidiary's pre-tax return on equity is anticipated to be 200% for the year, and its accounting balance sheet and income statement would appear as follows:

17. Assuming the firm receives full use of the proceeds of the short sale.

18. The option must be protected from dividend payouts.

19. We saw previously that the purchase of insurance was economically equivalent to the purchase of a put option on the net assets. That is also the case here since a European call option on the S&P 500 is equivalent to a European put option on a *short* position in the S&P 500, that is, a put option on the net assets.

20. Thus, the value of *perfect* stock-selection skills equals the value of the risk capital of the portfolio since *with such skills*, the portfolio *never* underperforms the index and its risk capital is thus reduced to zero. For a theory that equates the value of market timing to the value of a portfolio guarantee, see R.C. Merton (1981), "On Market Timing and Investment Performance Part I: An Equilibrium Theory of Value for Market Forecasts," *Journal of Business*, 54 (July): 363-406.

21. In October, 1987, prior to the stock market crash, the British government arranged to sell its $12.2 billion stake in British Petroleum to the public. The underwriting firms agreed to pay $65 per share, a full month before the offering would come to market. The shares fell to $53 post crash. According to *The New York Times*, October 30, 1987, the four U.S. underwriters collectively stood to lose in excess of $500 million. A subsequent price guarantee from the Bank of England reduced these losses to an estimated $200 million after tax.

**Allocating the costs of risk capital to individual businesses or projects is a problem for organizations that operate in a more or less decentralized fashion. Moreover, any allocation must necessarily be *imputed*, if only because highly risky principal transactions often require little or no up-front expenditure of cash.**

**ACCOUNTING BALANCE SHEET F**

| | | | |
|---|---|---|---|
| Net working capital | $10 | Shareholder equity | $10 |

**ACCOUNTING INCOME STATEMENT F**

| | |
|---|---|
| Revenues (underwriting spreads) | $50 |
| Customary expenses | (30) |
| Profit before tax | 20 |
| Pre-tax ROE | 200% |

This accounting analysis, however, ignores risk capital, which in this case is the price of the insurance (implicitly provided by the parent) needed to ensure that the subsidiary can perform its underwriting commitments. Suppose such insurance would cost $15 million in premiums. Then the risk capital balance sheet of the subsidiary would include the insurance as an asset, and total shareholder equity would be $25 million, consisting of $10 million of cash capital and $15 million of risk capital.

**RISK-CAPITAL BALANCE SHEET F**

| | | | |
|---|---|---|---|
| Net working capital | $10 | Cash capital | $10 |
| Underwriting guarantee (from parent) | 15 | Risk capital | 15 |
| Total Assets | 25 | Shareholder equity | 25 |

After the fact, if the underwriting business performs as anticipated, the parent guarantee will not have been needed. Thus, the insurance that enabled the subsidiary to get the business in the first place will have expired worthless. As shown below in Table F, including the cost of this insurance (which expired worthless) in the income statement results in an anticipated net profit of $5 million, or a pre-tax return of 20% on *economic* equity of $25 million:

**TABLE F**
**ANTICIPATED NET PROFIT INCLUDING RISK CAPITAL**

| | |
|---|---|
| Revenues (underwriting spreads) | $50 |
| Customary expenses | (30) |
| Underwriting insurance | (15) |
| Profit before tax | 5 |
| Pre-tax ROE | 20% |

The expensing of the $15 million cost of insurance shown in Table F is standard accounting practice if the insurance is obtained from arms-length providers. The fact that the parent provides the insurance should not change the treatment. Thus, the proper internal accounting would book the $15 million insurance premium as an expense to the underwriting subsidiary, and as revenue to the parent in its role as guarantor. Correspondingly, any "claims" paid on the guarantee should be considered revenue to the sub and an expense to the parent.

Even though this treatment of revenue and expense does not affect consolidated accounting, it can materially affect the calculated profit rates of individual businesses within the firm. In particular, the omission of risk capital "expended" on insurance overstates profits when the underlying assets perform well (because the insurance expires worthless) and understates profits when the underlying assets perform poorly (because the insurance becomes valuable).

## THE ECONOMIC COST OF RISK CAPITAL

Accounting for risk capital in the calculation of actual after-the-fact profits is important for reporting and other purposes, such as profit-related compensation. For the purposes of decision-making *before the fact*, however, *expected* profits must be estimated. This requires estimation of the *expected* or *economic* cost of risk capital. Since risk capital is used to purchase insurance, and insurance is a financial asset, risk capital will not be costly in the economic sense if the insurance can be purchased at its "actuarially" fair market value. For example, the purchase of $100 worth of IBM stock is not costly in this sense if it can be purchased for $100.

Usually, however, transacting is not costless. Typically, a spread is paid over fair market value. These spread costs are "deadweight" losses to the firm. In terms of traditional use of "bid-ask" spread, the bid price from the firm's perspective is the fair value and the ask price is the amount the firm must actually pay for the insurance. The *economic cost* of risk capital to the firm is thus the spread it pays in the purchase of this insurance.

The reasons for such spreads in insurance contracts vary by type of risk coverage, but the largest component for the type discussed here generally relates to the insurer's need for protection against various forms of information risks and agency costs:

- *Adverse selection* is the risk insurers face in not being able to distinguish "good" risks from "bad."

Unable to discriminate perfectly, they limit amounts of coverage and set prices based on an intermediate quality of risk, and try to do so to profit enough from the good risks to offset losses incurred in the underpricing of bad risks.[22]

- *Moral hazard* is the risk insurers face if they are not able to monitor the actions of the insured. Once covered, those insured have an incentive to increase their asset risk.
- *Agency costs* are the dissipation of asset values through inefficiency or mismanagement. As residual claimants with few contractual controls over the actions of the firm, equityholders bear the brunt of these costs.

Because principal financial firms are typically opaque in their structure, insurers of such firms—capital providers included—are especially exposed to these information and agency risks. Spreads for providing asset insurance to these types of firms—and hence their economic cost of risk capital—will therefore be relatively higher than for more transparent institutions.

The cost of risk capital is likely to depend on the form in which the insurance is purchased. The spreads on each form of insurance are determined differently. For example, in an all-equity firm, the required asset insurance is "sold" to the firm by its shareholders. The cost of risk capital obtained in this way will tend to reflect high agency costs (given the extensive leeway afforded to management by this structure), but little in the way of moral-hazard costs since there is no benefit to management or the firm's shareholders from increasing risk for its own sake. Debt financing, on the other hand, can impose a discipline on management that reduces agency costs. But then moral-hazard spreads can be high, especially in highly leveraged firms in which debtholders perceive a strong incentive for management to "roll the dice." The task for management is to weigh the spread costs of the different sources of asset insurance to find the most *efficient* way of "spending" the firm's risk capital.

Managing the firm most efficiently does not necessarily imply obtaining the lowest cost of risk capital. Consider the case of *signaling costs*. Firms faced with high spread charges can try to obtain lower spreads by making themselves more transparent, signaling that they are "good" firms. For example, "good" firms can report on a mark-to-market basis knowing that the cost to "bad" firms of doing so would be prohibitive (they would be seized by creditors and/or lose their customers). Transparency, however, can also impose costs of its own. For example, increasing transparency could lead to greater disclosure of proprietary strategies or self-imposed trading restraints that prevent it from taking advantage of short-lived windows of opportunity. Thus, the principal firm has to trade off between paying higher spread costs of risk capital for opaqueness and paying signaling costs and sacrificing potential competitive advantages to achieve transparency.

In calculating expected profitability for the overall firm, risk-capital costs should be expensed along with cash-capital costs. To illustrate, consider the example of Balance Sheet D in which the firm required $2.5 billion of cash capital and $500 million of risk capital. Because the cash capital is riskless, its cost is the AAAA rate (a little less than LIBOR), assumed to be 10% per annum. Suppose that the spread or economic cost of one-year risk capital for this firm is $30 million.[23] That is, the fair value "bid price" of the insurance provided by risk capital is $470 million and the "ask price" is $500 million. The $30 million spread is thus 6% of the ask price. Then total economic capital costs for the firm will be as follows:

| | | |
|---|---|---|
| Cash capital costs: | $250 | (10% of $2.5 billion) |
| Risk capital costs: | 30 | (6% of $500 million) |

The rate paid for cash capital is the same for all firms, the riskless rate, here 10%. Risk capital costs could vary considerably among firms, and in a few special cases they could be negligible.[24]

This example differs importantly from the previous securities underwriting example (Table F). In Table F, we deducted the full "premium" expended on the purchase of insurance, while here we consider only that portion of the premium attributable to the spread or economic cost. The full insurance premium is deducted when the purpose of the

22. For a general discussion of these risks and costs in the context of insurance contracts, see Karl H. Borch (1990), *Economics of Insurance*, Amsterdam: North-Holland.

23. For an explicit model of these spread costs, see Merton (1993), cited in note 1.

24. For example, an open-end mutual fund is highly transparent. Moreover, the liabilityholders are principally customers who can redeem shares daily. Enforced by the securities laws, the selection of assets matches the promised contingent payments on customer liabilities, as expressed in the fund's prospectus. Hence *net* assets are virtually riskless.

**The cost of risk capital is likely to depend on the form in which the insurance is purchased. The task for management is to weigh the spread costs of the different sources of asset insurance to find the most *efficient* way of "spending" the firm's risk capital.**

analysis is to measure profits after the fact, or *ex post.* But when the purpose of the analysis is to measure the cost of capital *ex ante*, only the economic cost is deducted because, *ex ante*, insurance purchased free of spread costs at its actuarial fair value is just that—costless.

We next apply our concept of risk capital to two important areas of firm management.

## HEDGING AND RISK MANAGEMENT

The implications of our framework for hedging and risk-management decisions are straightforward. Exposures to broad market risk—such as stock market risk, interest rate risk, or foreign exchange risk—usually can be hedged with derivatives such as futures, forwards, swaps and options. By definition, hedging away these risk exposures reduces asset risk. Thus, hedging market exposure reduces the required amount of risk capital.

Firms that speculate on the direction of the market, and therefore maintain a market exposure, will require more risk capital. By purchasing put options to insure against these market risks, the firm can maintain its desired exposures with the least amount of risk capital.

If there were no spread costs for risk capital, larger amounts of risk capital would impose no additional costs on the firm. In this case, firms may well be indifferent to hedging or not.[25] But if there are spread costs, and if these costs depend on the amount of risk capital, then a reduction in risk capital from hedging will lead to lower costs of risk capital if the hedges can be acquired at relatively small spreads.[26] That will usually be the case with hedging instruments for broad market risks where significant informational advantages among market participants are unlikely.[27]

## CAPITAL ALLOCATION AND CAPITAL BUDGETING

Financial firms frequently need to consider entering new businesses or getting out of existing businesses. The cost of risk capital can be a major influence on these decisions. As always, the marginal benefit must be traded off against the marginal cost. But to evaluate the net marginal benefit of a decision is difficult, because in principle it requires a comparison of total firm values under the alternatives being considered.

One simplifying assumption is that the incremental cost of risk capital is proportional to the incremental *amount* of risk capital. This might be reasonable, for example, if the decision does not lead to disclosures that materially change the degree of transparency or opaqueness of the firm. In this case, calculation of the economic cost of risk capital for a particular business is equivalent to the calculation of the risk capital applicable to that business.

Even if there are no economic costs of risk capital, calculation of the amount of risk capital of a particular business is still relevant. As discussed in example F, allocations of risk capital to individual businesses within the firm are necessary to calculate their after-the-fact profits. Such profit calculations can then serve, for example, as the basis for incentive compensation awards.

In general, the incremental risk capital of a particular business within the firm will differ from its risk capital determined on the basis of a stand-alone analysis. As we shall demonstrate, this results from a diversification effect that can dramatically reduce the firm's overall risk capital. The importance of this externality from risk-sharing depends on the correlations among the profits of the firm's various businesses. Its presence means that a full allocation of all the risk capital of the firm to its constituent businesses is generally inappropriate.

We illustrate with an example of a firm with three distinct businesses. Table 1 shows the current gross assets, customer liabilities, net assets (investor capital), and one-year risk-capital requirements of each business on a stand-alone basis.[28] The businesses all have the same amounts of gross assets, but different amounts of net assets because they have different amounts of customer liabilities. Business 1

25. Except if it changes the transparency or opaqueness of the firm, as discussed previously.

26. Merton (1993), cited in note 1, provides a model of spread costs that produces this result.

27. For example, for an explanation of the very narrow observed spreads on stock-index futures relative to the spreads on individual stocks, see James F. Gammill and and A.F. Perold (1989), "The Changing Character of Stock Market Liquidity," *The Journal of Portfolio Management*, 15 (Spring): 13-18.

28. Risk capital in this example is computed using the loan guarantee model in Merton (1977), cited in note 15, which is based on the Black-Scholes option-pricing model. Risk capital for this model will be roughly proportional to the standard deviation of profits. See the Technical Appendix for the precise calculations. See Merton (1993) and Merton and Bodie (1992), cited in note 1, for an extensive bibliography of more general models for valuing loan guarantees.

requires substantial amounts of investor capital but relatively little stand-alone risk capital. Business 3 is the riskiest, requiring the most stand-alone risk capital; however, it has the least investor capital. Business 2 is fairly risky and requires a moderate amount of investor capital.

**TABLE 1 ($ MILLIONS)**

| Gross Assets | Customer Liabilities | Investor Capital | Stand-Alone Risk Capital |
|---|---|---|---|
| BUSINESS 1 | | | |
| $1,000 | $500 | $500 | $150 |
| BUSINESS 2 | | | |
| 1,000 | 600 | 400 | 200 |
| BUSINESS 3 | | | |
| 1,000 | 700 | 300 | 250 |
| TOTAL | | | |
| $3,000 | $1,800 | $,1200 | $600 |

Table 2 shows how the profits of the three businesses are correlated. With a correlation coefficient of .5, the profit streams of Business 1 and Business 2 are fairly highly correlated. The profits of Business 3, by contrast, are completely uncorrelated with those of Businesses 1 and 2.

**TABLE 2: CORRELATION AMONG BUSINESSES**

| | Business 1 | Business 2 |
|---|---|---|
| Business 2 | .5 | |
| Business 3 | 0 | 0 |

Because the businesses are not perfectly correlated with one another, there will be a diversification benefit: the risk of the portfolio of businesses will be less than the sum of the stand-alone risks of the businesses. Risk capital—the value of insurance on the portfolio of assets—will therefore mirror this effect, and the risk capital for the total firm will be less than the sum of the (stand-alone) risk capital necessary to support each of the three businesses. For example, based on the correlations in Table 2, the risk capital of the firm can be shown to be $394 million, a 34% reduction relative to the aggregate risk capital on a stand-alone basis (see Technical Appendix).

The reduction in risk capital derives from the interaction among the risks of the individual businesses. The less-than-perfect correlation among their year-to-year profits leaves room for one business to do well while another does poorly. In effect, the businesses in the portfolio coinsure one another, thus requiring less external asset insurance.

An important implication of this risk-reduction effect is that businesses that would be unprofitable on a stand-alone basis because of high risk-capital requirements might be profitable within a firm that has other businesses with offsetting risks. Thus, the true profitability of individual businesses within the multi-business firm will be distorted if calculated on the basis of stand-alone risk capital. A decision-making process based on this approach will forgo profitable opportunities.

The alternative approach of allocating the risk capital of the combined firm across individual businesses also suffers from this problem. To show why, we examine the *marginal* risk capital required by a business. This can be done by calculating the risk capital required for the firm without this business, and subtracting it from the risk capital required for the full portfolio of businesses. Doing so for the three businesses in our example produces the results in Table 3:

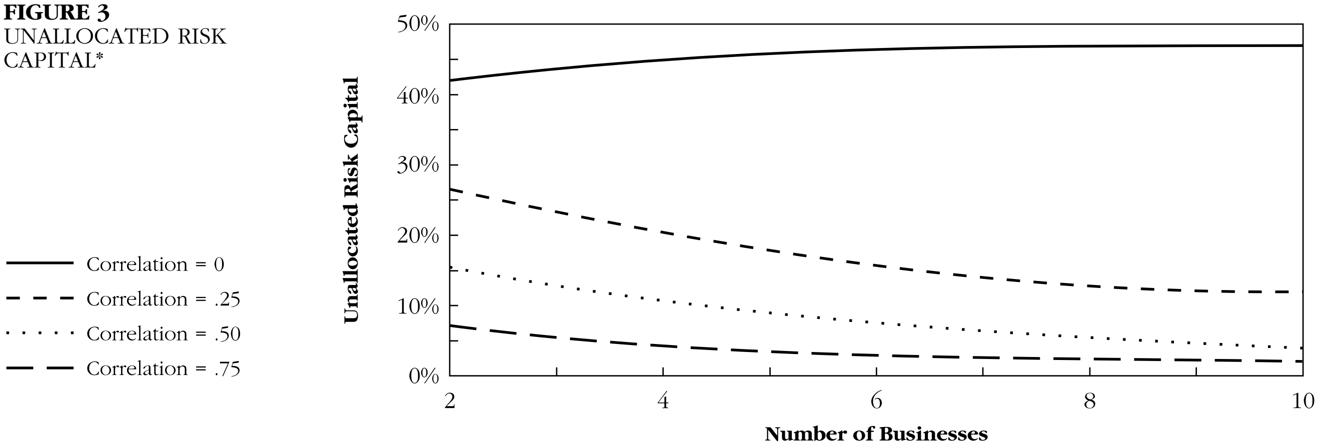
**TABLE 3**

| Combination of Businesses | Required Risk Capital for Combination | Marginal Business | Marginal Risk Capital |
|---|---|---|---|
| 1+2+3 | $294 | | |
| 2+3 | 320 | 1 | $74 |
| 1+3 | 292 | 2 | 102 |
| 1+2 | 304 | 3 | 90 |
| | Summation of marginals: | | $266 |

The first line of Table 3 shows the required risk capital for the combination of all three businesses, taking into account the less than perfect correlations among the businesses. As already noted, this amounts to $394 million. The next three lines of Table 3 show the calculation of the marginal risk capital of each business. For example, in the second line, we calculate the required risk capital for a firm composed of just businesses 2 and 3, taking into account the zero correlation between these businesses. It amounts to $320 million. The difference between $320 million and the required risk capital for all three businesses is $74 million. This is the marginal risk capital for business 1. It is the reduction in risk capital that a firm in businesses 1, 2, and 3 would achieve by exiting business 1; or it is the additional risk capital required for a firm in businesses 2 and 3 to enter business 1.

**Less-than-perfect correlations among the profits of a financial firm's different businesses can dramatically reduce the firm's risk capital. In effect, the businesses in the portfolio coinsure one another, thus requiring less external asset insurance.**

**FIGURE 3**
UNALLOCATED RISK CAPITAL*




*Percentage of total firm risk capital not accounted for by the marginal risk capital of the individual businesses. Calculations assume businesses are symmetrically correlated and have the same stand-alone risk capital.

For the purposes of making the marginal decision, the cost of marginal risk capital should be used. As shown in the last line of Table 3, however, the summation of marginal risk capital, $266 million, is only two thirds of the full risk capital of $394 million required for the firm. Thus, if marginal risk capital is used for allocation among businesses, $128 million (32% of total risk capital) will not be allocated to any business.[29]

The discrepancy between the total risk capital of the firm and the sum of the marginal risk capital of its businesses will of course depend on the specifics of those businesses, but it can be very large. Using the aggregate of marginal risk capital, Figure 3 illustrates how much of the firm's total risk capital goes unallocated as a function of the number of businesses in the firm, and the correlation among their profits. The analysis assumes that all businesses are the same size (in terms of stand-alone risk capital) and are symmetrically correlated. As shown in Figure 3, the unallocated capital is larger at lower correlations. Only at the extreme of perfect correlation among the businesses is all of the capital allocated. In all other cases, at least some is not allocated. In the case of no correlation among the businesses, for example, the marginal risk capital of the individual businesses can account for as little as 50% of firm risk capital, so that as much as 50% can (and should) go unallocated.

These conclusions hold quite generally. Full allocation of the firm's risk capital overstates the marginal amount of risk capital. And the risk capital of a business evaluated on a stand-alone basis overstates the marginal risk capital by an even greater amount.[30]

Taking into account correlations among profits of individual businesses in capital-budgeting analysis may seem at odds with the traditional CAPM-based notion that the only correlations that matter are those between individual business units and the broad market. Correlations among business units matter here because, by affecting the total amount of risk capital needed to support the businesses, they ultimately affect the total economic costs of risk capital. Per our earlier discussion, the economic cost of risk capital is the deadweight loss of spreads. The firm must expect to earn profits in excess of this cost as well as the cost of cash capital, which is the riskless rate of interest.

In traditional capital-budgeting procedures, estimates of cash flow correlations with the market portfolio are used to determine their stand-alone asset values. In our framework, these stand-alone asset values are assumed as given. Hence, correla-

29. "Grossing up" the marginal allocations (by 32 percent in the example) to "fully allocate" the firm's risk capital does not solve the problem. Instead, it overstates the benefits of reductions in risk capital from dropping businesses or not starting new ones.

30. See the Technical Appendix for a formal proof of these propositions. Merton (1993) provides another extensive example. The fact that risk capital cannot be allocated stems from the "externality" arising out of the less-than-perfect correlations among the profits of individual businesses and the asymmetric risk faced by providers of insurance: limited upside and potentially large downside.

tions of business units with the broad market enter only indirectly—that is, in determining the amount of risk capital. As an insurance premium, risk capital is a function of the riskiness of the net assets as well as the value of the net assets. The *riskiness* of the net assets is affected by correlations among business units; the *value* of net assets is affected by their correlation with the broad market.

## SUMMARY AND CONCLUSIONS

Financial firms that act as principal in the ordinary course of business do so in terms of asset-related as well as liability-related activities. Liability-related activities (such as deposit-taking and issuing guarantees like insurance and letters of credit) are mostly customer-driven, which makes such businesses credit-sensitive. Principal activities create a special set of financing, capital budgeting, and risk management decisions for the firm.

We have developed a framework for analyzing those decisions within the principal financial firm. The framework is built around a concept of risk capital, which we define as the smallest amount that can be invested to insure the *net* assets of the firm against loss in value relative to a risk-free investment. Using this definition of risk capital, the paper develops a number of important conclusions:

- The amount of risk capital is uniquely determined, and depends only on the riskiness of the net assets. It is not affected by the form of financing of net assets.
- Risk capital funds are provided by the firm's residual claimants, usually shareholders (except in the case of extremely highly leveraged firms). Implicitly or explicitly, this capital is used to purchase asset insurance. Potential issuers of asset insurance to the firm are third-party guarantors and the firm's stakeholders, including customers, debtholders, and shareholders.
- The economic costs of risk capital to the firm are the spreads on the price of asset insurance that stem from information costs (adverse selection and moral hazard) and agency costs.
- For a given configuration, the risk capital of a multi-business firm is less than the aggregate risk capital of the businesses on a stand-alone basis. Full allocation of risk-capital across the individual businesses of the firm therefore is generally not feasible. Attempts at such a full allocation can significantly distort the true profitability of individual businesses.

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## ■ TECHNICAL APPENDIX

### CALCULATION OF RISK CAPITAL IN TABLE 1 AND TABLE 3

For a given business, let the value of gross assets at time t be denoted by $A_t$, and the default-free value of customer liabilities be denoted by $L_t$, for $0 \leq t \leq T$. Gross assets and customer liabilities may both have uncertain, contingent payoffs. The value of the net assets at time t is $A_t - L_t$. If the net assets were invested risklessly, they would amount to $(A_0 - L_0)\exp(rT)$ at time T, where r is the continuously-compounded riskless rate of interest. The shortfall in net assets relative to a riskless return is thus $(A_0 - L_0)\exp(rT) - (A_T - L_T)$, so that insurance to permit default-free financing of the net assets must pay $\max\{(A_0 - L_0)\exp(rT) - (A_T - L_T), 0\}$ at time T. This is the same payoff structure as a European put option on the net assets with exercise price $(A_0 - L_0)\exp(rT)$. Under the assumption that the gross assets and customer liabilities both follow geometric Brownian motions, the value of this put option, and hence the amount of risk capital, is given by:

$$\text{Risk Capital} = A_0 F(1,1,0,T,\sigma)$$

where $F(S,E,r,T,\sigma)$ is the Black-Scholes (1973) formula[31] for a European call option on a stock with initial value S, exercise price E, riskless rate r, expiration date T, and volatility $\sigma$.[32] Here, $\sigma$ is the volatility of profits as measured by the volatility of percentage changes in the ratio of gross assets to customer liabilities $A_t/L_t$ (or simply the percent volatility of gross assets if customer liabilities are fixed or are non-existent.) As shown by Taylor's expansion for $\sigma\sqrt{t}$ not too large, the formula for risk capital is closely approximated by:

$$\text{Risk Capital} \approx .4A_0\sigma\sqrt{T}.$$

The formula used here for the variance rate of profits for a combination of N businesses is given by $\Sigma\Sigma w_i w_j \rho_{ij} \sigma_i \sigma_j$, where $\rho_{ij}$ is the correlation between the profits of businesses i and j, and $w_i$ is the fraction of gross assets in business i. The formula is an approximation that applies exactly only if investments in the businesses are continuously rebalanced so that the volatilities of the profits of the individual businesses maintain their relative proportions over the interval 0 to T. For the purposes here, this approximation has no material effect.

In Table 1, the volatility of business profits was assumed to be 37.5%, 50%, and 62.5% per annum, respectively. Using the above variance formula, the volatility of the profits of the combination of three businesses evaluates to 32.75% per annum. This low percentage volatility of the three businesses combined stems directly from the diversification effect. The pairwise combinations show a similar effect.

Table A shows that for the range of parameter values used here the approximation $.4A_0\sigma\sqrt{T}$ is very close to the exact Black-Scholes option value:

**TABLE A ($ MILLIONS)**

| Gross Assets | Standard Deviation ($\sigma$) | Approximate Risk Capital ($.4A_0\sigma\sqrt{T}$) | "Exact" Risk Capital (Black-Scholes) |
|---|---|---|---|
| BUSINESS 1 | | | |
| $1,000 | 37.5% | $150 | $148.7 |
| BUSINESS 2 | | | |
| 1,000 | 50.0% | 200 | 197.4 |
| BUSINESS 3 | | | |
| 1,000 | 62.5% | 250 | 245.3 |
| BUSINESSES 1+2 | | | |
| 2,000 | 38.0% | 304 | 301.4 |
| BUSINESSES 1+3 | | | |
| 2,000 | 36.4% | 292 | 288.8 |
| BUSINESSES 2+3 | | | |
| 2,000 | 40.0% | 320 | 317.0 |
| BUSINESSES 1+2+3 | | | |
| 3,000 | 32.8% | 394 | 390.8 |

31. Set forth in F. Black and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81 (May-June): 637-654.

32. $\sigma^2 = \sigma^2_A + \sigma^2_L - \sigma_A\sigma_L\rho_{AL}$, where $\sigma_A$ is the volatility of gross asset returns, $\sigma_L$ is the volatility of customer liability "returns," and $\rho_{AL}$ is the correlation between gross asset returns and customer returns. See Stanley Fischer, (1978), "Call Option Pricing When the Exercise Price is Uncertain, and the Valuation of Index Bonds," *Journal of Finance*, 33 (March): 169-176; William Margrabe, 1978), "The Value of an Option to Exchange One Asset for Another," *Journal of Finance*, 33 (March):177-186; and, especially, René Stulz, (1982), "Options on the Minimum or the Maximum of Two Risky Assets: Analysis and Applications," *Journal of Financial Economics*, 10 (July): 161-185.

## THE RELATIONSHIP OF MARGINAL RISK CAPITAL TO COMBINED AND STAND-ALONE RISK CAPITAL

This section establishes the general propositions that a) the sum of the risk capital of stand-alone businesses exceeds the risk capital of the businesses combined in one firm; and b) the risk capital of a combination of businesses exceeds the sum of the marginal risk capital of each of those businesses.

As in the first part of this Appendix, let $X = (A_0 - L_0)\exp(rT) - (A_T - L_T)$ be the shortfall (or surplus if it is negative) in the net assets of a business at time T. Let there be N individual businesses, and let $X_i$ be the shortfall for business i. From the above, insurance to permit default-free financing of the net assets of business i must pay $f(X_i) = \max\{X_i, 0\}$ at time T. Note that the function f(.) is convex and satisfies $f(0) = 0$.

The sum of the insurance payoffs to the stand-alone businesses is $\Sigma f(X_i)$, and the insurance payoff to the combined businesses is $f(\Sigma X_i)$. Since f(.) is convex, we can apply Jensen's inequality to obtain:

$$\Sigma f(X_i) \geq f(\Sigma X_i)$$

which establishes the first proposition.[33]

To establish the second proposition, we note that $f(\Sigma_{j\neq i} X_j)$ is the insurance payoff to the firm consisting of all businesses except i. Thus the marginal insurance payoff for business i is

$$f(\Sigma X_i) - f(\Sigma_{j\neq i} X_j).$$

We now observe the identity:

$$\Sigma X_i = \Sigma_i(\Sigma_{j\neq i} X_j)/(N-1).$$

Therefore, by Jensen's inequality,

$$f(\Sigma X_i) \leq \Sigma_i f((\Sigma_{j\neq i} X_j)/(N-1)).$$

Applying Jensen's inequality a second time and using the fact that $f(0) = 0$, we obtain

$$(N-1)f(\Sigma X_i) \leq \Sigma_i f(\Sigma_{j\neq i} X_j)$$

from which it follows that

$$Nf(\Sigma X_i) - \Sigma_i f(\Sigma_{j\neq i} X_j) \leq f(\Sigma X_i)$$

or

$$\Sigma_k\{f(\Sigma X_i) - f(\Sigma_{j\neq k} X_j)\} \leq f(\Sigma X_i).$$

This proves that the sum of the marginal insurance payoffs is at most the insurance payoff to the combined firm. Therefore, the risk capital of a combination of businesses exceeds the sum of the marginal risk capital of each of those businesses.

33. This is the well-known proposition that a portfolio of options always returns at least as much as the corresponding option on a portfolio of underlying securities.