Multi-Period Discounted Cash Flow Rate-making Models in Property-Liability Insurance

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Abstract

Discounted cash flow (DCF) models have become increasingly important in property-liability insurance pricing. This article analyzes the two most prominent DCF models—the Myers-Cohn (MC) model and the National Council on Compensation Insurance (NCCI) model. The MC model is shown to imply constant capital structure based on present value concepts, while the NCCI model implies constant capital structure based on book values of reserves and surplus. The models reflect alternative and potentially testable hypotheses regarding the timing of the equity flows involved in the insurance transaction. Because the equity timing differs, the models do not generally give the same results.

Introduction

Property-liability insurance contracts are characterized by a time lag between the premium payment and loss settlement dates. During this time lag, the insurer earns investment income on the unexpended component of the premium. Given this timing difference, it is surprising that the recognition of investment income in ratemaking is a relatively recent phenomenon. Prior to the late 1960s, property-liability ratemaking formulas included as a profit margin a flat percentage of the gross premium (usually 5 percent). Timing differences between premiums and claims and the resulting investment income were ignored in formal ratemaking procedures.[1]

During the late 1960s, rising claim costs and higher interest rates began to motivate regulators to scrutinize ratemaking formulas more carefully. The

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[1] Underwriting profits as high as 5 percent were rarely realized in practice. The industry maintained that the smaller realized returns implicitly recognized investment income, and there is substantial evidence of an inverse relationship between underwriting returns and investment yields [see Cummins and Nye (1980), Cummins and Harrington (1985), and Smith (1989)]. Nevertheless, the rationality of the decision process can only be improved by replacing pricing fictions with appropriate financial models.

result was that states such as New Jersey and Texas began to require insurers to give explicit consideration to investment income in ratemaking.

Since property-liability insurance involves cash flows at different points, the models developed in corporate finance would seem to have been the logical starting point for the recognition of investment income in insurance rates. However, property-liability insurance is deeply mired in statutory accounting. As a result, the earliest models were based on accounting concepts.

The accounting models have been extremely influential and are still proposed in some jurisdictions by insurers and their rating bureaus such as the Insurance Services Office (ISO). Descriptions and analyses of accounting models can be found in NAIC (1983), Cummins and Chang (1983), and Williams (1983). The most serious defects of the accounting models are that they are retrospective rather than prospective and they use embedded yields to measure the rate of return on policyholder funds. Both of these characteristics are contrary to well-known principles of corporate finance.[2]

To estimate the cash flows that will result from any given insurance policy, the accounting models look backward instead of forward. They typically measure policyholder funds as a proportion of unearned premiums and loss reserves. Policyholder funds are multiplied by the rate of return on the company's investment portfolio (the embedded yield) to obtain the investment income credit.

Reserves are an imperfect proxy, at best, for the amount and timing of future cash flows. (See Cummins and Chang (1983) for an analysis of this problem.) Reserves represent sunk costs which should be irrelevant in setting rates for policies issued in the future. They do contain some information on the time lag between the premium and loss payment dates, but this information can be easily extracted for use in a correct ratemaking formula.

The embedded yield is also irrelevant for ratemaking. The correct rate of investment return is the estimated rate that will be earned on the funds received under any given policy or policy cohort.[3] This has nothing to do with past investment yields. When the insurer receives the premium under a newly issued policy, these funds (net of expenses) will be invested at current market rates, not at the embedded yield. Hence, ratemaking should always reflect the best possible estimate of the yields that will be attainable when the cash flows are received.[4]

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[2] Accounting methodologies also are prominent in public utility regulation. For an evaluation, see Kolbe, Read, and Hall (1984).

[3] Insurers can hedge the risk that the rate of return will change between the date the premium is set and the date the funds are received by using futures contracts and other hedging mechanisms. The feasibility of hedging may be affected in some cases by regulatory restrictions. In such cases, the risk loading should reflect the risk of possible variations in interest rates.

[4] The accounting methodologies also attempt to provide the insurer with a reward for risk-bearing based on a more rational foundation than the traditional flat percentage. One approach is to compare the mean and variance of book returns across industries. This is sometimes called the comparable earnings method. Insurance usually plots below the "market line" derived from these statistics. The usual inference is that insurance is under-earning for an industry in its risk class [see NAIC (1987) and Arthur D. Little (1967)]. Interindustry book return comparisons

The recognition of these and other defects has led to the development of more appropriate financial pricing models for property-liability insurance contracts. An excellent review of these models is presented in D'Arcy and Doherty (1988). Among the methods proposed for insurance pricing are the capital asset pricing model (CAPM) [e.g., Biger and Kahane (1978) and Fairley (1979)], the option pricing model (OPM) [e.g., Doherty and Garven (1986)], arbitrage pricing theory [Kraus and Ross (1982)], and more general models of continuous time finance [Cummins (1988a)]. These models and their successors will eventually provide the basis for sophisticated financial pricing of property-liability insurance contracts. Presently, however, due to measurement problems and other difficulties, none of these models is fully operational, at least for purposes of application in a regulatory context.

The financial models that have gained the most prominence as regulatory ratemaking methodologies are discrete-time discounted cash flow (DCF) models.[5] Unlike the accounting models, DCF models are soundly based in financial theory. The estimation and application of these models is much more manageable than that of the continuous time models mentioned above. In addition, DCF models have a straightforward intuitive interpretation. These models have significant potential for use by regulators as well as by underwriters in pricing long-tail lines of insurance. In effect, they provide a method for underwriters to gauge the reasonableness of investment income credits when evaluating competing rate offers. They could be easily incorporated into underwriting work-stations using conventional spreadsheet programs.

This article presents and analyzes the two most prominent DCF models that are now used in property-liability ratemaking. The models were developed by Myers and Cohn (1987) and by the National Council on Compensation Insurance (NCCI). The Myers-Cohn (MC) model is used in regulating automobile and workers' compensation insurance rates in Massachusetts and has been proposed in other states. The NCCI uses its model in regulatory rate filings in several jurisdictions. Although these models are rapidly becoming the predominant method of property-liability insurance ratemaking, there has been very little discussion of them in either academic or actuarial literature.

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are invalid because they rest on the assumption that there should be a positive relationship between book return and book variance. This is wrong on at least two counts—market rather than book values represent the correct basis for comparison and the systematic component(s) of total variance will be priced differently from the unsystematic component. Even if book figures were appropriate for evaluating insurance risk and return, the inter-industry method would be suspect due to differences in accounting procedures across industries. For example, the book values of assets for most industrial firms tend to be less than their market values. Insurers, on the other hand, invest primarily in marketable securities, which have book values in most periods that are closer to their market values. Thus, other things equal, an industrial firm will show a higher book return on equity than an insurer. Fisher and McGowan (1983) have shown that even intra-industry comparisons can be misleading if firms utilize different depreciation methods.

[5] The term discounted cash flow (DCF) model is typically used to refer to a specific method for estimating the cost of capital (the Gordon-Shapiro growth model). The term DCF is used here in a more generic sense.

D'Arcy and Doherty (1988) provide a brief discussion of discounted cash flow models but do not specifically analyze either the NCCI or the MC model. The Myers-Cohn model was introduced in Myers and Cohn (1987), but its economic implications were not extensively discussed in that source. The present article provides such a discussion. It also provides the first published presentation and analysis of the NCCI model. In addition, since both models were developed prior to the enactment of the Tax Reform Act of 1986, formulas are developed for the losses-incurred and underwriting expense deductions under the 1986 Act and these formulas are incorporated into the models.[6]

On the surface, the Myers-Cohn and NCCI models differ significantly; on closer inspection, however, the two models are not entirely dissimilar. However, there are important differences between the two models that must be clearly understood if they are to be used correctly. Besides providing an economic analysis of the two models, this article indicates the conditions under which they yield consistent results.

The principles of DCF modeling as they apply in insurance are discussed first. A clear statement of these principles is important because they are so often violated in regulatory proceedings. Then an economic analysis of the models based on a two-period insurance policy is provided. The two-period case permits focusing on the economic interpretation of the models while avoiding the complications that are necessarily present in a multi-period context. The models are reconciled with each other and with corporate capital budgeting analysis. The multiple period DCF models are then presented and numerical examples are given based on an actual workers' compensation rate filing, followed by conclusions and proposed directions for future research.

Principles of Insurance DCF Modeling

The MC model and the NCCI model are based on concepts of capital budgeting. Essentially, the insurance policy is viewed as a project under consideration by the firm. The methods can be used to arrive at a price (the premium) for the project that will provide a fair rate of return to the insurer, taking into account the timing and risk of the cash flows from the policy as well as the market rate of interest.[7]

Given the parallels with capital budgeting, it is tempting simply to adopt an "off-the-shelf" model from one of the leading corporate finance textbooks. This approach would overlook critically important subtleties of the insurance transaction and could lead to serious errors in computing insurance premiums. Such errors are rather common in rate regulatory proceedings.

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[6] Programs to compute these deductions also have been developed by other researchers and research organizations such as the NCCI. These programs are not currently available in the published literature.

[7] In its applications of the model, the NCCI uses the model to determine whether a particular set of proposed rates will generate a fair internal rate of return. This is a slightly different orientation from using the model to establish the rates.

This section discusses the principles of insurance DCF modeling and points out common pitfalls.

General Principles

Most of the common errors can easily be avoided by following a few basic principles, which are similar to the principles of capital budgeting set forth in finance texts [e.g., Brealey and Myers (1988) and Ross and Westerfield (1988)]. The DCF principles that are most important in insurance are value-additivity, irrelevance of accounting, and avoidance of double counting.

Value additivity. Each policy (or policy cohort in the case of ratemaking for a state) should stand on its own. In particular, prices should not reflect the insurer's past experience, such as embedded yields and sunk costs.

Irrelevance of accounting (both statutory and GAAP). Accounting numbers are not relevant in a DCF analysis except as they directly impact cash flows. There are few direct feedbacks from a firm's accounting numbers to its cash flows.[8] Where feedbacks exist, the appropriate approach is to quantify their effects on the firm's cash flows. The existence of feedbacks from accounting is not a valid justification for ignoring financial principles and adopting an accounting methodology. Among other things, the irrelevance of accounting implies that loss reserves and loss development factors are irrelevant in DCF analysis. Instead, the analysis should focus on the loss payout pattern so that a paid loss rather than an incurred loss triangle should be used. The use of paid development is consistent not only with principles of corporate finance but with the international actuarial literature [Taylor (1986)].

Avoidance of double counting. Double counting is always a potential problem in capital budgeting but seems to be particularly significant in insurance. The way to avoid double counting is to *adopt a perspective.* Two perspectives are available, both of which can lead to correct results: the policy can be priced from the point of view of either the policyholder or the company.[9] Flows from one are flows to the other. Thus, if appropriately defined the two perspectives lead to models that are mirror images of one another. Mixing the perspectives can lead to double counting.

Insurance Cash Flows

The MC model defines flows from the policyholder perspective, while the NCCI model utilizes the company perspective. The relevant flows appear in Table 1.

The MC approach counts all flows that the policyholder either pays or receives. Thus, the policyholder pays premiums (a cash outflow) and receives

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[8] An important exception is the firm's Federal tax payments, which to some extent are based on accounting results. Regulatory restrictions that affect cash flows also may be triggered by accounting reports. The rule to keep in mind is that the cash flow or change in cash flow is important, not the accounting trigger.

[9] Actually, prices will be established in most cases for a policy block or cohort. In any event, the flows are understood to be expected values.

Table 1

Perspectives In Insurance Cash Flow Analysis

Policyholder Perspective (MC)	Company Perspective (NCCI)
Premium Payments (net of expenses)	Surplus Commitment
Loss Payments	Investment Income (net of taxes)
Corporate Taxes	Underwriting Profit (loss)
	Release of Surplus Commitment

loss payments (cash inflow). The policyholder is also responsible for paying the corporate taxes that result from the insurance transaction. These include both taxes on underwriting profits and taxes on investment income received on funds backing the policy, including the unexpended *premium balance* (which is defined as premiums received less cumulative losses paid) and the surplus committed to the policy.[10]

Under MC, surplus is assumed to be committed in proportion to premiums when the policy is initiated and to be released to the company as losses are paid. The investment income tax on this surplus must be paid by the policyholder in order for the firm to receive a fair rate of return. The rationale is that the owners of the firm could invest directly in financial assets and not have to pay the corporate income tax. They will not subject themselves to an additional layer of taxation by investing in an insurer. Therefore, the policyholder must pay the corporate tax [Myers and Cohn (1987, pp. 57–58)]. This is the only direct compensation the company receives for its surplus commitment under the MC model. Because surplus is committed when the policy is written (an outflow for the company) and released as the losses are paid (an inflow), the MC model is also sometimes called the *surplus flow* model, although this term could as easily be applied to the NCCI model.[11]

The NCCI model also recognizes the company's surplus commitment. In this model, it is the surplus flow itself and not just the taxes on the surplus account (as in MC) that becomes a discounted cash flow. A surplus outflow (inflow) also occurs if the policy is written at an underwriting loss (profit). The timing of the underwriting loss (profit) flow is an important issue, which

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[10] This is not meant to imply that either premiums or losses paid are accumulated at interest in the premium balance. This balance is the sum of premiums received less the sum of losses paid.

[11] A variant of the Myers-Cohn model sometimes proposed in regulatory proceedings is the surplus block model, where all surplus is released at the end of the accident (or policy) year. This, of course, reduces the indicated premium. The surplus block model implies that no surplus backing is needed during the reserve runoff period, i.e., that all uncertainty necessitating surplus backing is resolved after all premiums are earned. This would seem to be very difficult to justify in long-tail lines where the proportion of unsettled claims, including those incurred but not reported (IBNR), at the end of the accident (policy) year is very high.

is discussed in more detail below.[12] The NCCI model assumes that the company receives the investment income (net of taxes) on the surplus commitment and premium balance. At first glance, the use of investment income as a flow seems to be contrary to capital budgeting principles. However, as shown below, this is not a problem if it is done correctly.

An issue in both models is the appropriate level of surplus commitment. Usually, this commitment is based on industry-wide ratios of premiums-to-surplus (MC) or reserves-to-surplus (NCCI). The reason for this is that no widely-accepted theoretical rationale for surplus commitment in insurance presently exists. Ultimately, the level of surplus is determined by supply and demand considerations and perhaps by regulation. For example, prices set in a competitive insurance market can be expected to lead to an equilibrium "demand for solvency," which would imply a level of surplus commitment. The capital costs required to maintain this surplus commitment would be included in the price of insurance. The more advanced financial pricing models mentioned above [e.g., Doherty-Garven (1986) and Cummins (1988a)] provide the foundations for a theory of surplus commitment. However, until the details of this theory have been worked out, a market premiums-to-surplus or reserves-to-surplus ratio will continue to be used. The following analysis assumes the existence of such a ratio, imposed by regulatory fiat. This assumption is consistent with the realities of insurance ratemaking in a regulatory environment.

Internal Rate of Return (IRR) v. Net Present Value (NPV)

As in conventional capital budgeting, an important issue in insurance DCF analysis is whether to utilize an internal rate of return (IRR) or net present value (NPV) approach. The NCCI model uses IRR, while Myers-Cohn use NPV.

The problems with the IRR method are well-known [see, for example, Brealey and Myers (1988, pp. 77–85)]. Two IRR pitfalls often lead to errors in insurance applications.

Consider the formula for the net present value (NPV) of a cash flow stream:

$$\mathrm{NPV} = C_0 + C_1/(1+R) + C_2/(1+R)^2 + C_3/(1+R)^3 + \ldots \qquad (1)$$

where C_i = cash flow at time i, and
R = the discount rate.

The IRR is defined as the discount rate that results in NPV = 0. In the usual corporate capital budgeting problem, C_0 is negative (the cost of the project) and the later cash flows are positive (net returns from the project). In this case, the intuitive interpretation is that the firm is lending C_0 and receiving returns from the loan in later years. The firm would like the rate of return on

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[12] Underwriting loss flows occur after the underwriting balance becomes negative. The underwriting balance is defined as the sum of premiums received less the sum of losses paid and taxes paid (or tax credits received). Thus, tax credits are considered in arriving at the amounts of the underwriting loss flows, but the loss flows themselves are not directly reduced to reflect taxes.

a loan to be as large as possible, hence, the decision rule is to accept if IRR is greater than R_C, where R_C equals the cost of capital appropriate for the project.

An "off-the-shelf" approach to insurance pricing, suggested in some rate hearings, views the problem from the company perspective, considering premiums as inflows and losses as outflows. While this is not necessarily incorrect, it can be misleading in an IRR context because the signs of the flows are opposite to those in the usual capital budgeting problem; the flows are positive initially and negative later on.[13] The intuitive interpretation is that the firm is borrowing rather than lending. It would like the cost of borrowing to be as small as possible. Thus, the decision rule is to accept if IRR is less than R_C.

Similar ambiguities arise in interpreting changes in the cash flow stream under this off-the-shelf approach. For example, an increase in the premium (C_0) leads to a reduction in the IRR, while a proportionate increase in every loss flow leads to an increase in IRR. While the IRR still leads to correct decisions if interpreted correctly, the seemingly counterintuitive nature of the relationships can present significant problems in a regulatory setting. As in finance generally, using the NPV approach is advisable.

Two-Period Insurance DCF Models

The distinctions between the models can be seen most easily by considering a simple two-period case where the premium is received at time 0 and losses are paid at time 1. Taxes are assumed to be paid at time 1. Surplus is assumed to be committed at time 0 at surplus-to-premium ratio δ (MC) or surplus-to-reserves ratio ϕ (NCCI). It is assumed that assets are invested at the risk-free rate (R_f). The importance of this assumption is analyzed in a subsequent section.

The MC Model

The MC model solves the following equation for P, the premium:[14]

$$PV(P) = PV(L) + PV(Tax) \tag{2}$$

where PV(.) = the present value operator,
P = the premium,
L = expected loss payment, and

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[13] This is not necessarily the case. The NCCI model, for example, is formulated from the perspective of the equity provider. In the NCCI's application of the IRR model, it is assumed that the insurer must make an equity commitment equal to the underwriting loss early in the policy period. Under these circumstances, the early flow are likely to be negative, paralleling the usual capital budgeting example.

[14] Underwriting expenses are omitted to simplify the exposition. In practice, they could be considered essentially riskless and netted out of the premium flows.

Table 2

Cash Flows in Two-Period Myers-Cohn Model

Flow	Time: 0	1	Discount Rate
Premium	P	0	R_f
Loss	0	L	R_L
Underwriting Profits Tax	0	τ(P-L)	R_f, R_L
Investment Balance (IB)	P(1 + δ)	0	
IB Tax	0	$\tau R_f P(1+\delta)$	R_f

Key: P = premiums, L = expected losses, R_f = risk-free rate of interest, R_L = risk-adjusted discount rate for losses, τ = corporate income tax rate, δ = surplus-to-premiums ratio.

TAX = corporate taxes on underwriting and investment income associated with this policy or policy block. Relevant investment income is investment income on the premium balance (cumulative premiums less cumulative losses and expenses) and on committed surplus.

The method is an application of Myers' adjusted present value method [see Brealey and Myers (1988, pp. 443–446)]. The project is evaluated as if it were totally equity financed, and each component of the cash flow stream is evaluated at the risk-adjusted discount rate appropriate for that flow.

The cash flows for the two-period MC model are presented in Table 2. The risk-adjusted discount rate for loss flows is R_L. Although any theoretically defensible risk adjusted discount rate (RADR) could be used, the model is usually applied using the CAPM: $R_L = R_f + \beta_L[R_m - R_f]$, where β_L has the obvious definition [see Myers and Cohn (1987)].

Using the cash flows in Table 2, equation (2) becomes:

$$P = L/(1+R_L) + \tau R_f P(1+\delta)/(1+R_f) + \tau P/(1+R_f) - \tau L/(1+R_L) \quad (3)$$

Notice that the premium component of the underwriting profit tax is discounted at the risk-free rate, while the loss component is discounted at the RADR for losses. Solving equation (3) for P and simplifying yields:

$$P = \frac{L}{(1+R_L)(1-\tau R_f \delta/[(1+R_f)(1-\tau)])} \quad (4)$$

The comparative statics are as follows: $P_{RL} < 0$, $P_\tau > 0$, $P_\delta > 0$, and $P_{Rf} < 0$, where subscripts indicate partial derivatives with respect to the subscripted variable or parameter.[15] The denominator of equation (4) is a

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[15] It is possible to construct a set of parameter values such that P_{Rf} would be greater than 0. However, such a combination of parameter values would not be economically meaningful.

Table 3

Cash Flows in Two-Period NCCI Model

Flow	Time: 0	1	Discount Rate
Surplus	$-\phi D$	ϕD	R
Investment Income	0	$(1-\tau)(P+\phi D)R_f$	R
Underwriting Profit	0	$(1-\tau)(P-L)$	R

Key: P = premiums, L = expected losses, R_f = risk-free rate of interest, R_L = risk-adjusted discount rate for losses, τ = corporate income tax rate, ϕ = surplus-to-reserves ratio, D = reserves.

tax-adjusted RADR analogous to Fairley's (1979) tax-adjusted underwriting margin. However, his result gives an underwriting profit margin with a k (reserves-to- premiums) factor to represent the payout tail, while this discount rate is applied directly to the loss cash flow.

The NCCI Model

The cash flows for the NCCI model are shown in Table 3. As mentioned above, this is usually applied as an IRR model. Hence, one would solve for R (the IRR) such that the present value of all cash flows is zero. The result is then compared with the cost of capital R_C. The fair premium is the value for which IRR equals R_C. As discussed above, it would be more appropriate to use the cost of capital as a discount rate and set the premium set so that NPV equals 0.

For comparison with the MC model, it is helpful to set the present value of the NCCI cash flows equal to zero and solve for P, given Rc. The result is equation (5):

$$P = \frac{L + \phi D[\tau R_f + (R_C - R_f)]/(1-\tau)}{1 + R_f} \tag{5}$$

where D = reserves, and

ϕ = the surplus-to-reserves ratio.

In general, the formulas (4) and (5) do not give the same results. Two conditions under which they do give the same results are: (1) If all flows are discounted at the same rate, i.e., if $R_f = R_C = R_L$, or (2) if, in the NCCI model, losses are discounted at R_L and all other flows at R_f. The comparability of the models is discussed in more detail below.

Evaluation

MC are applying orthodox capital budgeting theory. Their model implies that the cost of insurance is the present value of losses, discounted to reflect systematic risk, plus the present value of the corporate taxes incurred as a result of pooling risks through a corporate insurance entity. The NCCI model is conceptually quite similar to MC. The premium in the NCCI model is the present value of expected losses; plus an amount sufficient, after-tax, to pay the taxes on investment income on committed surplus; plus an after-tax risk charge (R_C-R_f), also proportional to committed surplus.

Since the models are usually applied using CAPM risk-adjusted discount rates or costs of capital, no loading usually is present for unsystematic risk, i.e., the probability of ruin is not priced. However, neither model is specifically linked to the CAPM. Any defensible cost of capital or RADR formula could be used.

The MC model compensates the policyholder at the risk-free rate for the loss of use of funds between the premium and loss payment dates. To facilitate comparability, the NCCI model was derived above on the assumption that investments are in risk-free assets. However, in actual applications of this model, the anticipated market rate of investment return usually is applied.

The use of the risk-free rate is based on Fairley's argument that the policyholder does not buy insurance to take investment risk. Consequently, if the company chooses to invest premiums and surplus in riskier assets, it should bear the risk and receive the return (or loss) from the riskier strategy. According to Fairley, the policyholder is insulated from the company's investment risk-taking behavior through the regulatory requirement that investment income credits follow the risk-free rate.

Recent work by Cummins (1988a) reveals that Fairley's argument is incorrect and that the investment income credit should be at the company's anticipated market return, not at the risk-free rate. If policyholders are credited with investment income at R_f and if guaranty fund premiums are not risk- based, the insurer has an incentive to pursue risky investment strategies, which increase the probability of bankruptcy. There is no market penalty for doing this since the existence of the guaranty fund renders all policies free of default risk.[16] If guaranty fund premiums are flat, there is no penalty from the guaranty fund either, and all gains from risky investing accrue to the company. If the company must credit the policyholder with investment income based on its anticipated portfolio mix, the problem is mitigated because part of the gain from risky investing accrues to the policyholder.

Cummins (1988b) develops risk-adjusted discount rates for risky insurance policies, i.e., policies issued by firms that can become bankrupt. Under these circumstances, the following risk adjusted discount rate would be appropriate:

$$R_L = R_f + [(D_A A/D)\,\beta_A + (D_L L/D)\,\beta_L]\,[R_m - R_f] \tag{6}$$

where A = assets,
L = liabilities,
D = the value of debt, i.e., $L\,[\exp(-R_f T) - B(x,T)]$,

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[16] Obviously, there is an economic cost to being a policyholder or claimant of a defunct insurance company, even in the presence of a guaranty fund. However, the fund alters the cost calculus by significantly reducing the penalty.

$B(x,T)$ = a Black-Scholes put option on the firm's asset-to-liability ratio, $x = A/L$, with exercise price 1 and time to expiration T, and

D_A, D_L = partial derivatives of D with respect to A and L, respectively.

In this formula, the risk adjustment would be a function of the firm's capital structure and its asset risk, as well as the liability risk. The additional risk charge would either reduce the policyholder's premium, if no guaranty fund were present, or would be paid to the guaranty fund. This would eliminate the firm's incentive to take unnecessary investment risk. To simplify the notation and facilitate comparison with prior work, the remainder of the article adheres to the convention of risk-free investment and the standard CAPM-based RADR.

Reconciliation of the Models

As mentioned above, the MC and NCCI models generally do not produce the same result. To explore the issue of model consistency in more detail, it is revealing to compare the rate of return on shareholders' equity under the MC and NCCI premium formulas. The year-end values for the two models are defined as:

$$\text{MC: } V_1 = P(1+R_f) + \delta P(1+R_f) - L - \tau(P-L) - \tau R_f P(1+\delta) \qquad (7a)$$

$$\text{NCCI: } V_1 = P(1+R_f) + \phi D(1+R_f) - L - \tau(P-L) - \tau R_f(P+\phi D) \qquad (7b)$$

In equations (7a) and (7b), V_1 can be thought of as the value of the stockholders' equity at time 1 if the company wrote only this policy. The stockholders' return is defined as $V_1/S-1$, where S is initial surplus commitment. Calculation reveals the following rates of return on the company's surplus commitment:

$$\text{MC: } R = R_f + (L/S)(1-\tau)(R_f-R_L)/(1+R_L) \qquad (8a)$$

$$\text{NCCI: } R = R_C \qquad (8b)$$

The MC result has a clear intuitive interpretation. Capital is rewarded at the profit margin (R_f-R_L) on the basis of the present value of the losses assumed by the company, $L/(1+R_L)$.[17] The value of losses is after-tax, reflecting the Federal tax shield for loss payments. The intuitive interpretation of the NCCI model is also clear: the company is compensated for its surplus commitment at the IRR.

To see the relationship between the two models, consider the formula for the cost of capital of a levered firm [e.g., Ross and Westerfield (1988, p. 355)]:

$$R_C = R_A + (D/S)(R_A-R_L) \qquad (9)$$

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[17] The term (R_f-R_L) is positive if the loss beta, β_L, is negative. Most estimates indicate that β_L is negative [Fairley (1979) and Cummins and Harrington (1985)]. However, positive loss betas were recorded by Cummins and Harrington (1985) in some periods.

where D = the market value of liabilities,
S = market value of equity (surplus), and
R_A = the rate of return on the firm's assets.

Using (9) with R_f substituted for R_A, it is clear that (8a) and (8b) will be equivalent if D, the market value of liabilities, is equal to $L(1-\tau)/(1+R_L)$. The latter expression is the market value of liabilities for the insurer assuming that losses generate a tax shield that can be immediately recovered at full value.

These results indicate that both the MC and NCCI models give correct results if interpreted correctly. However, the results imply that the present market value, not the book value, of liabilities must be used in the NCCI approach. Thus, the surplus-to-reserves ratio should be based on the estimated market value of liabilities, not the book value.[18]

The NCCI model has a possible advantage over the MC model in terms of parameter estimation. This is the case because the cost of capital R_C is easier to estimate than the liability beta β_L, at least for traded firms [see Cummins and Harrington (1985)]. However, the use of a company-wide cost of capital implicitly assumes that the new policy has the same risk-return characteristics as the firm as a whole. Although this assumption may be questionable in multiple line companies, the error involved by assuming equivalent risk may be less than the error that would be introduced by using a liability beta. The development of divisional (i.e., by line) costs of capital for an insurance company is an unresolved issue that would be a fruitful topic for future research.

Multiple Period Insurance DCF Models

Although based on the same principles, the versions of the MC and NCCI models actually used in ratemaking cover multiple periods and are inevitably more complex than the two-period models discussed above. The formulas and assumptions used in the multiple period models are presented in Tables 4 through 7. The discussion proceeds by first presenting some basic multi-period theory. Next, the Federal income tax components of the ratemaking formulas are discussed, followed by numerical examples based on an actual workers' compensation rate filing.

Some Multi-Period Theory

When the analysis is extended to multiple periods, the differences between the two methods emerge more clearly. The MC model is considered first. The equation that would be solved to obtain the MC premium for the case where cash flows occur one and two periods from the present is shown below:

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[18] The results also suggest that an adjustment may be appropriate if the tax shield cannot be immediately recovered with certainty. The "tax option" discussed in Cummins and Grace (1988) would be useful in making this adjustment.

Myers-Cohn Premium:

$$P = L\left[\frac{C_1}{1+R_L} + \frac{C_2}{(1+R_L)^2}\right] + \frac{\tau\delta PR_f}{1-\tau}\left[\frac{1}{1+R_f} + \frac{C_2}{(1+R_f)^2}\right] \quad (10)$$

where C_j = the proportion of total losses paid at time j. To simplify the discussion, all premiums are assumed to be paid at time 0. The Myers-Cohn premium has two components: (1) the present value of losses, which are discounted at R_L, and (2) the present value of an amount sufficient to pay the tax on committed surplus ($\delta P = S$), discounted at R_f.

The underwriting profit tax is implicitly allowed for but simplifies out in deriving equation (10). An intuitive interpretation is that because the insurer is taxed on both the premium and the interest on the premium, the present value of the taxes on P is τP. Thus, the net (after-tax) value of the premium is ($1-\tau$). Because each loss term is also multiplied by the tax multiplier ($1-\tau$), the multiplier cancels out of the premium and loss terms in deriving equation (10). The surplus tax term nominally depends upon P in (10), but what actually matters is just the amount of surplus. It would perhaps be more direct to replace $P\delta$ by S in (10) and make an analogous change in the NCCI formulas.

The shareholder flows under the Myers-Cohn model are shown in equations (11a) and (11b):

Myers-Cohn Shareholder Flows:

$$Y_0 = -\delta P = -S \quad (11a)$$

$$Y_i = (1-\tau)R_f\, IB_{i-1} + (P-L)c_i(1-\tau) + P\delta c_i \quad (11b)$$

where Y_i = shareholder flow at time i.

Equation (11a) shows that the shareholders are required to put up surplus at time 0. This is an outflow from the shareholder perspective. Equation (11b) shows that the surplus flows back to the shareholders at rate c_i at period i. All surplus eventually is returned (in an expected value sense) because the c_i sum to 1. The shareholder flows also include the underwriting profit (loss), which in total is equal to $(P-L)(1-\tau)$. The assumption is that the underwriting loss flow is realized as losses are paid, i.e., at rate c_i. The shareholders also receive after-tax investment income on the investment balance, which is the sum of the policyholder and shareholder funds backing the policy.

The Myers-Cohn investment balance is shown as equation (12):

Myers-Cohn Investment Balance:

$$IB_i = P\left[\sum_{j=0}^{i} p_j - \sum_{j=0}^{i} c_j + \delta\left(1 - \sum_{j=0}^{i} c_j\right)\right] \quad (12)$$

The investment balance consists of accumulated premiums, less accumulated loss payments, plus the remaining surplus balance.

Equation (12) reveals an important feature of the Myers-Cohn model — the ratio of policyholder to shareholder funds remains constant throughout the policy payout period. To see this, assume that all premiums are paid at time 0 so that the investment balance is:

$$IB_i = P\left[(1 - \sum_{j=0}^{i} c_j) + \delta(1 - \sum_{j=0}^{i} c_j)\right] \tag{12a}$$

The first parenthetical term inside the brackets in equation (12a) represents policyholder funds, i.e., the unreleased premium balance. The second term represents shareholder funds, i.e., unreleased surplus. Considering the policyholder funds as analogous to debt and the shareholder funds as equity, the MC assumption is that leverage (capital structure) remains constant throughout the policy period at debt to equity ratio $1/\delta$. This result is not changed if premiums are paid in installments. The interpretation in that case is that premiums are being financed at the risk-free rate. Thus, the balance sheet still carries the same amount of debt (policyholder funds) but there is an offsetting asset item, premiums receivable.

The NCCI premium equation, shareholder flows, and investment balance are presented as equations (13), (14), and (15):

NCCI Premium:

$$\begin{aligned} P\left[1 + \frac{R_f\tau}{1+R} + \frac{R_f\tau c_2}{(1+R)^2}\right] \\ = L\left[1 - \frac{R_f(1-\tau)}{1+R} - \frac{R_f(1-\tau)c_2}{(1+R)^2}\right] \\ + \frac{S}{1-\tau}\left[1 - \frac{c_1}{1+R} - \frac{c_2}{(1+R)^2} - \frac{(1-\tau)R_f}{1+R} - \frac{(1-\tau)R_f c_2}{(1+R)^2}\right] \end{aligned} \tag{13}$$

NCCI Shareholder Flows:

$$Y_0 = -\phi L + (P-L)(1-\tau) \tag{14a}$$

$$Y_1 = (1-\tau)R_f\, IB_{i-1} + \phi L c_i \tag{14b}$$

NCCI Investment Balance:

$$IB_i = P\sum_{j=0}^{i} p_j - L\sum_{j=0}^{i} c_j - \tau(P-L)\sum_{j=0}^{i} c_j - (P-L)(1-\tau) + L\phi(1 - \sum_{j=0}^{i} c_j) \tag{15}$$

To understand the NCCI approach, it is best to start by discussing equations (14a), (14b) and (15). Like MC, the NCCI recognizes the commitment of surplus to the policy (policy block). In the NCCI case, this is ϕL (which could simply be set to S). Unlike MC, the NCCI assumes that the insurer also commits an amount equal to the after-tax underwriting loss $(P-L)(1-\tau)$. This illustrates an important difference between the two models. In MC the underwriting loss cash flows are assumed to occur as losses are paid (see the $(P-L)c_i$ terms in (11b)). In the NCCI model, on the other hand, the underwriting loss cash flow is assumed to occur at time 0. The change in the timing of this important flow implies that the models will not in general yield the same results in the multi-period case.

A second important difference between the two models has to do with the investment balance. Under both models, the after-tax investment income on the investment balance accrues to the shareholders. Under MC, the investment balance is simply the accumulated premiums, less accumulated losses, plus the remaining surplus commitment (see equation (12)). The NCCI investment balance incorporates the MC terms plus the after-tax underwriting loss plus the accumulated underwriting tax credits, $\tau(P-L)\Sigma c_j$. The underwriting loss component of the investment balance is initially set up net of tax credits. It is less than the amount needed to fully pay the underwriting loss by the amount of the tax credits. As the tax credits flow in (at the assumed rate c_j), they gradually offset this unfunded portion of the underwriting loss. The investment balance throws off after-tax investment income, which flows to shareholders (equation (14b)).

The NCCI investment balance at time 0 contains enough money, together with the tax credits and premiums that will be received, to pay all losses and return the company's surplus commitment. The MC investment balance is not adequate to pay all losses, even considering unpaid premiums and tax credits.[19] The deficit, the underwriting loss, is assumed to be funded in proportion to loss payments. The MC underwriting loss flows are offset against investment income (see equation (11b). This offset makes sense since the primary reason for the underwriting loss is to give policyholders credit for the investment income earned on their funds held by the company.

In the NCCI premium equation (equation (13)), the left hand side is the income due to the premium, including the premium itself and the investment income earned on the premium component of the surplus contribution for the underwriting loss. The present value of the premium flows (the left hand side of (13)) are set equal to the right hand side. The first term on the right hand side of (13) is the loss part of the surplus commitment for underwriting losses less the present value of the after-tax investment income on this surplus commitment. The last term in (13) is the surplus commitment, less the present value of surplus returns (inflows), less the present value of investment income on the surplus account.

In terms of a priori reasoning, the MC model stands up quite well. MC counts most flows as they are realized. Premiums and surplus are set aside to back the policy, and these amounts are drawn down proportionately as losses are paid (i.e., as the company is released from its loss liability). Profits are also taken as the loss liability is released. This assumes that the company earns the profit and receives the surplus outflow as it fulfills its promise to pay the losses, a result with considerable intuitive appeal. The balance sheet of the firm under MC would maintain a constant ratio of policyholder to shareholder funds.

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[19] This discussion assumes that the premium will be less than the sum of undiscounted losses and expenses, as is usually the case. At low interest rates or high risk loadings, this relationship may not hold.

The NCCI model assumes that the company fully funds the underwriting loss at policy inception. The NCCI maintains that its approach is consistent with the realities of a regulated insurance market where loss liabilities must be fully funded at nominal values.

Like MC, the NCCI model can be interpreted as giving rise to a constant leverage ratio. Consider equation (15) and assume that all premiums are paid at time 0 (recall that this assumption does not affect the leverage ratio). Then, interpret the underwriting loss (P-L) as policyholder funds and the funded tax credit on the underwriting loss, $\tau(P-L)(1-\Sigma c_j)$, as shareholder funds. The policyholders' claim on the firm under this interpretation is $L(1-\Sigma c_j)$ and the shareholders' claim is:

$$\tau(P-L)(1-\sum_{j=0}^{i} c_j) + L\phi(1-\sum_{j=0}^{i} c_j)$$

Since each term in both the policyholders' and shareholders' claims involves a constant multiplied by $(1-\Sigma c_j)$, a constant ratio between these amounts is maintained throughout the policy period.

The NCCI reserve is based on nominal losses, while the MC reserve is based on premiums, a present market value concept. Hence, the MC model approximates a constant market value capital structure, while the NCCI model represents a constant statutory book value capital structure. The MC model thus is closer to being consistent with the principles of modern financial theory than the NCCI model.

In a regulated industry, there may be a justification for departing from financial theoretic principles provided that the departures realistically reflect the impact of regulation on the firm's market value. The incorporation in a pricing model of accounting conventions that are disregarded by the market in establishing the firm's value would not be justifiable. The issue is whether or not the requirement that the company set up nominally valued reserves actually affects the market value of the firm; and, if so, whether its effect is captured accurately by the NCCI model. An investigation of this issue would be beyond the scope of this study. However, one must remark that it is not unknown for insurers to be significantly underreserved for sustained periods of time without incurring regulatory intervention. It seems unlikely that the statutory reserve constraint is as stringent as the NCCI model assumes.

Federal Income Tax Formulas

The Tax Reform Act of 1986 significantly changed the tax rules for property- liability insurers. The provisions of the Act are explained in Gleeson and Lenrow (1987a and b) and in U.S. House of Representatives (1986). Although the Act has wide-ranging effects on insurance company taxation, two provisions are especially important for DCF ratemaking: loss reserve deductions and underwriting expense deductions. These rules are discussed below and incorporated in the multiple period models. Other rules, such as proration, which taxes part of "tax exempt" interest and intercorporate

Table 4

Loss and Expense Deductions Under 1986 Tax Code

Years From Start of AY	**Deduction**
	LOSS DEDUCTIONS:
1	$d_1 = cy_1 + \sum_{i=2}^{Y} cy_i/(1+R_T)^{i-1.5}$
$j \geq 2$	$d_j = R_T \sum_{i=j}^{Y} cy_i/(1+R_T)^{i-j+.5} + cy_j\,[1 - (1+R_T)^{.5}]$
	EXPENSE DEDUCTIONS:
1	$de_1 = E - .2\,P[1 - \sum_{j=0}^{3} y_j]$
2	$de_2 = .2\,P \sum_{j=4}^{7} y_j$

KEY: d_i = the loss deduction for year i, R_T = the required annual interest rate for discounting losses under the 1986 Tax Reform Act, and cy_j = the expected proportion of total losses paid during year j as determined under the 1986 Tax Reform Act (these proportions generally will differ from cya_j, the actual proportions paid in year j), de_i = the expense deduction in year i, and y_j = proportion of premiums earned (losses incurred) in period j. This table assumes that all premiums are earned within two tax years of policy issue. The expense formulas reflect the 20 percent disallowance of expenses due to additions to the unearned premium reserve. Y = number of years in runoff period (the number of years for tax purposes may differ from the actual runoff period, see U.S. House of Representatives [26]). P = total undiscounted premiums, and E = total undiscounted expenses.

NOTE: The expense deductions are assumed to sum over quarters. Quarters 0 through 3 are in the first year after policy issue, while quarters 4 through 7 are in year 2. To simplify the notation, the loss deductions are annual. In practice, and in Tables 7 and 8, these annual deductions are allocated across the year on a quarterly basis.

dividends, also have implications for ratemaking by affecting investment strategy and hence the rate of investment return earned by insurers [Cummins and Grace (1988)]. These rules are beyond the scope of this study.[20]

The formulas for incurred loss and underwriting expense deductions are presented in Table 4. The loss and expense deductions are denoted d_i and de_i, respectively. Prior to the 1986 Tax Reform Act, insurers were permitted to deduct the full amount of losses and expenses incurred, even though a large proportion of losses would not be paid until long after the close of the tax year and even though a portion of expenses are prepaid, i.e., are attributable to revenues for the following year. The 1986 Act changed these deductions by permitting insurers to deduct only the present value of losses incurred and

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[20] The Tax Reform Act of 1986 has other important implications for insurers. For example, the Act may affect the use of policy year vs. accident year ratemaking and the manner in which policy year losses are allocated across accident years. Insurers also have the opportunity to choose their own payout pattern rather than the payout pattern promulgated by the IRS. These and other issues would need to be investigated to provide a comprehensive picture of the implications of the Act.

requiring the deferral of the prepaid expense deduction to match the corresponding revenues.

The prepaid expense deferral was approximated as 20 percent of the change in the unearned premium reserve. For an individual policy, this translates into the formulas for de_i in Table 4. The formulas assume one-year policies so that premiums are fully earned within two tax years. They also assume that all expenses are paid during the year of policy issue. This is done only to simplify the discussion and could easily be modified to reflect different payment patterns. Based on these assumptions, the deduction in the year the policy is written is equal to expenses paid, less 20 percent of unearned premiums at the end of the year. The latter component (20 percent of first year unearned premiums) becomes the deduction for the second year after policy issue. Thus, a full deduction is permitted but part is deferred to the second year. The company loses interest earnings on the deferral.

The loss deduction is more complicated. To simplify the discussion the assumption is made that all losses are incurred during the year the policy is issued.[21] During the year of issue, the company is allowed a full deduction for the amount assumed to be paid that year and, in addition, deducts the present value of the amounts to be paid in the future. The amounts paid each year are the company's estimate of total, undiscounted incurred losses, times payout proportions. For most companies, the long-tail payout proportions for tax purposes are estimated from the industry-wide Schedule P as given in *Best's Aggregates and Averages*. Thus, the payout proportions used for tax purposes generally differ from the payout proportions used to define the loss cash flows, which are estimated more precisely based on the company's own cash flow experience. Thus, the loss deduction allowed by the Code is only an approximation to the actual present value of losses.

The losses for any given tax year also give rise to deductions in subsequent tax years because the present value approach essentially disallows in the first accident-year the deduction for the component of loss payments that will be funded through investment earnings. This investment income component is deductible in subsequent years when the losses are paid (or, more precisely, assumed to be paid on the basis of the industry-wide cash flow estimates used by the Code). The deductions in Table 4 for years two and the following years reflect this offset.

Consider, for example, the deduction for the second year after policy issue, d_2. The company is allowed to deduct interest on the discounted loss deduction from the prior year (the summation term in d_1). However, crediting a full year's interest to the amount paid in year two (cy_2) would overshoot this

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[21] If this is not the case, the formulas in Table 4 apply to the amount of losses incurred in the year of issue. An analogous formula applies to losses incurred in the following year. Thus, for policies issued in a given base year, it is necessary only to determine the proportion of losses incurred in the base year and the following year and then apply the appropriate present value formula.

deduction. Consequently, a half year's interest on this amount is subtracted (the second term on the right hand side of d_2).

It can be shown that the sum of d_i over all i equals 1, i.e, the company ultimately is permitted to deduct 100 percent of incurred losses. The difference from the prior tax rules is that the timing of the deduction is changed.

Another point to notice regarding the loss deduction is that the present value generally will be calculated using a different interest rate than the other parts of the DCF formulas. The Tax Code specifies that the "Federal mid-term rate" will be used. In practice, the discount rate is promulgated for each tax year by the Internal Revenue Service. The promulgated rate applies to that particular year in perpetuity, i.e., the rate used in obtaining the deductions arising from a given tax year will not change in the future to reflect changes in market interest rates. The loss and expense deductions are, of course, the same in both the NCCI and MC models.

Numerical Examples

This section provides numerical examples of the MC and NCCI models based on data from an actual workers' compensation rate filing. Before presenting the examples, some additional details of the models are discussed.

The MC Model. The numerical example of the MC model is based on the multiple period premium formula presented in Table 5. The key assumptions for both this model and the NCCI model are shown in Table 7. The assumptions are designed to provide convenient and reasonable approximations of real world cash flow amounts and timing. For example, flows are assumed to be paid at mid-period. The following discussion focuses on assumptions and procedures that have significant theoretical implications.

The MC model solves for the premium such that the present value of premium flows equals the present value of loss flows, expense flows, and tax flows. As in the two-period case, each flow is discounted at the appropriate flow- specific discount rate (R_f for riskless flows and R_L for risky loss flows). Two tax flows are present: the underwriting profits tax [PV(Tax)] and the investment balance tax. To simplify the presentation in Table 5, the underwriting tax flow was assumed to occur at year end. In practice and in the numerical examples presented below, the company would make quarterly estimated tax payments.

The model assumes that the policy is for one year and is issued at the beginning of the year (see Table 7). Thus, all premiums are earned during the first year; and all expenses under the policy, which are assumed to be paid during the first year, are also deductible during that year. The loss deductions are based on the formulas in Table 4. The investment balance is the surplus commitment plus the net premium balance. In the spirit of the MC multi-period theory, premiums net of expenses (P-E) are assumed to flow into the premium balance. This amount is drawn down in proportion to loss payments. For ease of comparison with the NCCI model, the surplus

Table 5

Premium Formulas for the Myers-Cohn Model

MYERS-COHN MODEL:

$$Px_f(p_N) = L\,x_L(c_N) + \tau(P\text{-}E)R_f x_f(IB_{N-1}) + Ex_f(e_N) + PV(TAX)$$

DEFINITIONS:

$PV(TAX) = \tau\{(P\text{-}E)/(1+R_f)^4 - L[d_1/(1+R_L)^4 + \sum_{i=2}^{Y} d_i/(1+R_L)^{4i}]\}$

IB_i = investment balance at time i in Myers-Cohn model,

$= \sum_{j=1}^{i} (p^*_j - c_j) + \delta(1 - \sum_{j=1}^{i} c_i)$

$x_h(v_k) = \sum_{i=1}^{k} v_i/(1+R_h)^{i-1/2}$ = present value operator

$x_h(v_k)$ = the discounted present value at rate R_h of cash flow proportions v_i over the period 1 through k.

v_i = a "generic" cash flow proportion, where $\sum_{i=1}^{K} v_i = 1$ and K is the period of the last flow,

c_i = proportion of expected claims paid in period i,

e_i = proportion of expenses paid in period i,

p^*_i = proportion of premiums net of expenses (P-E) paid in period i,

p_i = proportion of premiums paid at time i,

τ = Federal corporate income tax rate,

δ = surplus-to-(net)premium [S/(P-E)] ratio,

R_f, R_L = quarterly risk-free and risk-adjusted discount rates,

d_i = loss deduction in period i under 1986 Tax Code (Table 4),

P, E, L = nominal values of premiums, expenses, and losses,

N = number of quarters in the runoff period, and

Y = number of years in the tax runoff period.

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commitment in the MC numerical example is based on the same proportion of loss liabilities as in the NCCI case, rather than on premiums. The funds in the investment balance (unreleased premiums plus unreleased surplus) earn interest at R_f, and this interest is taxed at rate τ, making up the investment balance tax.[22]

MC commit 100 percent of surplus when the policy is written, i.e., at the beginning of the policy period. An alternative approach would be to commit surplus as coverage is provided, i.e., as premiums are earned or losses are incurred. The MC approach would be correct if the insurer must put up the entire surplus commitment in order to issue the policy. For example, if the contractual agreement is viewed as the insurer's obligation to pay all losses

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[22] The tax credits that partially offset this tax under the 1986 Tax Code are part of the underwriting profits tax term in Table 5.

Table 6

Premium Formulas for the NCCI Model

NCCI MODEL:

$$0 = x_c(S_N) + [x_c(SBm_N) + x_c(UBm_N)]\,R_f(1-\tau)$$

DEFINITIONS:

S_o = surplus flow at time 0 = $-\phi L + (P-L-E)(1-\tau)$,
S_i = surplus flow at time i = $\phi L c_i$,
SB_i = surplus balance at time i,
= $\phi L(1 - \sum_{j=1}^{i} c_j)$
SBm_1 = $\phi(SB_i + SB_{i-1})/2$ = average surplus balance in period i,
UB_i = cumulative underwriting (premium) balance in period i
= $Px_0(p_i) - Ex_0(e_i) - Lx_0(c_i) - T_1 I_4 + L\tau \sum_{j=2}^{Y} d_j I_{4*j} - (P-L-E)(1-\tau)$,
I_k = 1 in quarters k and after, 0 otherwise,
T_1 = tax payment at end of year 1
= $\tau[P - E - Ld_1]$,
UBm = $(UB_i + UB_{i-1})/2$,
R_C = cost of capital,
ϕ = surplus-to-reserves ratio,
N, Y = number of quarters in the actual runoff period and number of years in tax runoff period, respectively,
$x_h(v_k)$ = $\sum_{i=1}^{k} v_i/(1+R_h)^{i-1/2}$ = present value operator.

arising under the policy and the policy is non-cancellable by the insurer, then it would make sense for all surplus to be committed at policy inception even though no losses have been incurred at that point. Committing all surplus at policy inception leads to higher premiums than if surplus were committed in proportion to incurred losses. Ultimately, both the amount and timing of the surplus commitment are endogenous to the market and possibly subject to binding regulatory constraints.

The NCCI Model. The multiple period NCCI model is presented in Table 6. This model is conceptually the same as the two-period model, i.e., the discounted flows are surplus and investment income. The company is assumed to commit surplus at time 0, and surplus flows back to the company as losses are paid.[23] After-tax investment income accrues to the company owing to the

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[23] In most actual applications, the NCCI model is applied to a block of policies covering a particular policy year. In this case, surplus is committed gradually over the course of the year. The gradual commitment of surplus is appropriate since the company typically would have the option to stop writing and renewing policies part way through the policy year. This contrasts with the

Table 7

Assumptions in Multiperiod Models

1. Flows are quarterly and occur at mid-quarter, except for tax flows, which occur at year-end. In reality, companies pay taxes quarterly based on estimated taxes for the tax year. This refinement was not included in the formulas in order to simplify the notation. However, quarterly tax payments are used in the numerical examples presented in Tables 7 and 8. In the numerical examples, the tax payout tail is assumed to be identical to the loss payout tail.
2. The model is for a one-year policy beginning at time 0. Thus, all premiums are earned by time 1. However, premium inflows can occur at any time.
3. All tax credits are fully recoverable at the time they occur. I.e., if taxable income is negative at any given time, the assumption is that it can be used to recapture past tax payments at that time.
4. Losses are assumed to be "risky" and thus discounted at a risk-adjusted discount rate. Expenses and premiums are assumed to be riskless and are discounted at the risk-free rate. (This assumption applies only to the MC model.)
5. All expenses are paid during the first four quarters. Hence, expenses are fully deductible in the first year after the policy is issued. The 1986 Federal Income Tax Code requires that the expense deduction be reduced by 20 percent of the change in the unearned premium reserve. The formulas could easily be modified to take this rule into account, using the results presented in Table 3.

investment of the premium (underwriting) balance (UB_i) and the surplus balance (SB_i). The underwriting tax flows, which are the same as in the MC case, enter the analysis through their impact on the underwriting balance, which affects the amount of investment earnings. This reflects the use of the company perspective in the NCCI model.

As mentioned above, the NCCI model assumes that that insurer funds the underwriting loss out of surplus at policy inception, whereas the MC model funds the underwriting loss as loss payments are made. This is a significant difference between the two models, which is reflected in the numerical examples given below. The NCCI approach is correct only if this treatment of surplus is an economic reality, i.e., if writing a block of policies requires the firm to completely forego the use of surplus equal to the underwriting loss early in the policy period rather than funding the loss more gradually out of investment income as losses are paid (the MC assumption). This depends upon the stringency of regulatory reserving and premiums-to-surplus constraints.

Examples. Numerical examples of the NCCI and MC cash flow models are presented in Tables 8 and 9, respectively. The cash flows appearing in these tables are based on an actual workers' compensation rate filing. They have been simplified for ease of presentation. For example, in the filing there are separate flows for allocated and unallocated loss adjustment expenses and for

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application of the model to a single policy, where the company does not have cancellation rights. In the latter case, as mentioned above, surplus should be committed at the policy issue date. For purposes of this example, both the NCCI and MC models are applied to a policy rather than a policy year.

premium taxes. The loss adjustment expenses have been folded into losses [column (3)], and all other expenses (underwriting expenses and premium taxes) are presented in column (2). Consistent with this particular filing, quarterly data are presented for the first two years following policy issue, and annual data are used thereafter. The annual tax flows for the first two years following policy issue have been spread evenly throughout these years. To clarify the losses incurred-deduction and avoid introducing two payout tails, the models are computed on the assumption that the tax payout tail and the loss payout tail are identical. The key assumptions are set forth in Table 7.

The approach in comparing the models is to calculate the premium using the MC model in Table 9. This premium is then inserted in the NCCI model (Table 8), and the latter is solved to yield the NCCI IRR.

Beginning with the NCCI case (Table 8), premiums, less expenses, losses paid, and taxes, constitute the underwriting flow. Column 5 in Table 8 is the underwriting flow plus the insurer's contribution of surplus for the underwriting loss. The underwriting loss surplus commitment (also called the "cash equity" commitment) takes place at time 0. The cumulative underwriting flow plus the cash equity constitute the accumulated underwriting account [column (6)]. This account eventually runs off to zero as all obligations under the policy are discharged.

It may appear that the underwriting loss flows should be multiplied by (1-τ) to reflect the loss tax shield. This would not be correct because the full amount of the tax shield has already been taken into account in computing the underwriting balance. Thus, the negative surplus flows for underwriting losses have already been reduced by the appropriate tax credits generated by the 1986 Tax Code.

For comparability with the MC example, the NCCI model is applied here as if the flows apply to a single policy issued at time zero. Surplus is committed to the policy at contract inception at the rate of one-third of expected losses and gradually returned to the insurer as losses are paid. The surplus balance (supporting surplus) is shown in column (8). Changes in the surplus balance plus the commitment of cash equity constitute the surplus flow [column (12)]. Increases in the balance are negative flows from the perspective of the capital providers and decreases are positive flows. Investment income is earned on the average surplus balance [column (9)] and average underwriting balance present during any given period. The net cash flow to capital providers [column (13)] is the after-tax investment income [columns (10) and (11) times (1-τ)] plus the surplus flow.

The internal rate of return (IRR) is the rate that discounts the net cash flows to zero. (The discounted values of the net cash flows are shown in column (15).) In this example, premiums equal \$103,616, undiscounted losses equal \$100,000, and expenses equal \$13,729. The policy has an IRR of 10.6 percent and a nominal underwriting loss of 9.8 percent of premiums.[24]

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[24] The nominal underwriting loss is (premiums - total losses - expenses)/premiums. An alternative definition would be to include tax credits in the numerator. This would be a more

Table 8

NCCI Cash Flow Model Example

RT = Tax Discount Rate = 7.0% Tax Rate = 34.0% IRR = 10.6%
R = Rate of Investment Return = 7.0% Surplus/Reserves = 0.33 PV(Net Cash Flow) = 0
Nominal UW Loss = -9.8%

Quarter	Premiums (1)	Expenses (2)	Expected Loss (3)	Fed Tax Flow (4)	Total UW Flow (5)	Accum UW Acct (6)	Loss Reserve LI-LPAID (7)	Suppt. Surplus (8)	Average Surplus (9)	Invest Income: On Surp (10) R*(9)	Invest Income: On UW (11) R*(6)	Surplus Flow (12)	Net Cash Flow (13)	Discount Factor (14)	Disc. NCF (15) 13*14
					6675			33333				-40008	-40008	1.0000	-40008
1	25904	-5958	- 1100	-1115	17731	15540	23900	32967	33150	580	272	367	929	0.9875	917
2	25904	-2590	- 3000	-1115	19199	34005	45900	31967	32467	568	595	1000	1768	0.9630	1702
3	25904	-2590	- 3900	-1115	18299	52754	67000	30667	31317	548	923	1300	2271	0.9391	2133
4	25904	-2590	- 4500	-1115	17699	70753	87500	29167	29917	524	1238	1500	2663	0.9158	2439
5	0	0	- 6100	332	- 5768	76719	81400	27133	28150	493	1343	2033	3245	0.8931	2898
6	0	0	- 4300	332	- 3968	71851	77100	25700	26417	462	1257	1433	2568	0.8709	2237
7	0	0	- 3500	332	- 3168	68284	73600	24533	25117	440	1195	1167	2245	0.8493	1907
8	0	0	- 3200	332	- 2868	65266	70400	23467	24000	420	1142	1067	2098	0.8282	1737
12	0	0	-13800	1054	-12746	57459	56600	18867	21167	1482	4022	4600	8233	0.7778	6403
16	0	0	-10000	844	- 9156	46509	46600	15533	17200	1204	3256	3333	6277	0.7034	4415
20	0	0	- 7500	694	- 6806	38528	39100	13033	14283	1000	2697	2500	4940	0.6362	3143
24	0	0	- 5800	584	- 5216	32517	33300	11100	12067	845	2276	1933	3993	0.5753	2297
28	0	0	- 4300	505	- 3795	28012	29000	9667	10383	727	1961	1433	3207	0.5203	1669
32	0	0	- 3900	443	- 3457	24386	25100	8367	9017	631	1707	1300	2843	0.4706	1338
36	0	0	- 3500	385	- 3115	21100	21600	7200	7783	545	1477	1167	2501	0.4256	1064
40	0	0	- 3000	335	- 2665	18210	18600	6200	6700	469	1275	1000	2151	0.3849	828
44	0	0	- 1900	300	- 1600	16077	16700	5567	5883	412	1125	633	1648	0.3481	574
48	0	0	- 2000	274	- 1726	14415	14700	4900	5233	366	1009	667	1574	0.3148	496
52	0	0	- 1500	252	- 1248	12928	13200	4400	4650	326	905	500	1312	0.2847	374
56	0	0	- 2600	221	- 2379	11114	10600	3533	3967	278	778	867	1563	0.2575	403
60	0	0	- 2200	179	- 2021	8914	8400	2800	3167	222	624	733	1291	0.2329	301
64	0	0	- 1900	143	- 1757	7026	6500	2167	2483	174	492	633	1073	0.2106	226
68	0	0	- 1600	111	- 1489	5403	4900	1633	1900	133	378	533	871	0.1905	166
72	0	0	- 1300	84	- 1216	4051	3600	1200	1417	99	284	433	686	0.1722	118
76	0	0	- 1100	62	- 1038	2924	2500	833	1017	71	205	367	549	0.1558	85
80	0	0	- 900	42	- 858	1976	1600	533	683	48	138	300	423	0.1409	60
84	0	0	- 700	26	- 674	1210	900	300	417	29	85	233	308	0.1274	39
88	0	0	- 400	15	- 385	681	500	167	233	16	48	133	176	0.1152	20
92	0	0	- 300	8	- 292	342	200	67	117	8	24	100	121	0.1042	13
96	0	0	- 100	3	- 97	147	100	33	50	4	10	33	42	0.0942	4
100	0	0	- 100	1	- 99	49	0	0	17	1	3	33	36	0.0852	3
						-0						0	0		0
TOTALS	103616	-13729	-100000	3438	- 6675					13123	32744	- 6675	63605		40008

Note: Column (4), Federal Tax Flow is calculated using the formula for T in Table 5. Column (5) = Total Cash Flow From Underwriting = Sum of columns (1) through (4) plus "cash equity," which equals the after-tax underwriting loss and is contributed at time zero. Column (6) = Accumulated underwriting account = Column (6) lagged 1 period + Average of (Column (5) and Column (5) lagged one period). Column (7) = Outstanding Losses = Losses Incurred - Losses Paid. One-fourth of total losses is incurred in each quarter of the first year. Column (8) = Supporting Surplus = (surplus/reserves) ratio * (total expected losses (100,000) minus cumulative losses paid). Column (9) = Average of Column (8) and Column (8) lagged 1 period. During the first two years, in columns (10) and (11), investment income per quarter is computed by multiplying the investment balances by R/4, to approximate quarterly return. During subsequent periods, investment return is credited at R. Column (12), Surplus Flow, is -1 times the change in Supporting Surplus [change in column (8)]. Column (14), Discount Factor, is computed at the IRR, on the assumption that payments are made at the midpoint of each period.

There are several important points to note. Tax credits serve to reduce premiums (raise the IRR). The implicit assumption is that all tax shields can be immediately recovered in full when they are generated. In reality, the company may not be able to recover all tax credits or may recover them only after a deferral period. Thus, the model gives the benefit of the doubt to the policyholder. The cash flows cover a period of 25 years. This rather long period is typical of workers' compensation; other coverages, such as auto bodily injury liability, would have shorter payout periods. For example, in Massachusetts, the auto liability payout period is eight years. The premiums that solve the model are very sensitive to the assumptions. Slight changes in the tax rate, the surplus-to-reserves ratio, and other parameters can have major

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accurate representation of reality than the commonly used measure of nominal underwriting result.

Table 9:

Myers-Cohn Cash Flow Model Example

RF = Risk Free Rate = 7.0%
RL = Risk Adj Disc Rate = 5.2%
RT = Tax Discount Rate = 7.0% Tax Rate = 34.0% PV(Net Cash Flow) = 0
R = Investment Return = 7.0% Surplus/Reserves = 0.33 Nominal UW Loss = -9.8%

Quarter	Premiums (1)	Expenses (2)	Expected Loss (3)	Loss Deductions (4)	Und Prof Tax Flow Riskless Part (5)	Und Prof Tax Flow Risky Part (6)	Invest Income: On Surp (10)	Invest Income: On UW (11)	Tax on Inv Inc (12)	Riskless Discount Factor (13)	Risk-Adj Discount Factor (14)	PV of Riskless Flow (15)	PV of Risky Flow (16)	Disc Net Cash Flow (17)
1	25904	-5958	- 1100	- 19193	- 7640	6526	580	166	- 254	0.9916	0.9937	11951	5391	17342
2	25904	-2590	- 3000	- 19193	- 7640	6526	568	512	- 367	0.9749	0.9812	14923	3459	18382
3	25904	-2590	- 3900	- 19193	- 7640	6526	548	866	- 481	0.9586	0.9688	14564	2544	17107
4	25904	-2590	- 4500	- 19193	- 7640	6526	524	1208	- 589	0.9425	0.9566	14218	1938	16155
5	0	0	- 6100	- 978	0	332	493	1328	- 619	0.9267	0.9446	- 574	- 5448	- 6022
6	0	0	- 4300	- 978	0	332	462	1247	- 581	0.9112	0.9327	- 529	- 3700	- 4230
7	0	0	- 3500	- 978	0	332	440	1185	- 552	0.8959	0.9209	- 495	- 2917	- 3412
8	0	0	- 3200	- 978	0	332	420	1133	- 528	0.8809	0.9093	- 465	- 2608	- 3073
12	0	0	- 13800	- 3101	0	1054	1482	3996	-1862	0.8444	0.8810	- 1572	-11229	-12801
16	0	0	- 10000	- 2483	0	844	1204	3247	-1513	0.7891	0.8374	- 1194	- 7667	- 8861
20	0	0	- 7500	- 2043	0	694	1000	2696	-1257	0.7375	0.7960	- 927	- 5417	- 6344
24	0	0	- 5800	- 1719	0	584	845	2278	-1062	0.6893	0.7567	- 732	- 3946	- 4678
28	0	0	- 4300	- 1485	0	505	727	1960	- 914	0.6442	0.7193	- 588	- 2730	- 3318
32	0	0	- 3900	- 1302	0	443	631	1702	- 793	0.6020	0.6837	- 478	- 2364	- 2842
36	0	0	- 3500	- 1134	0	385	545	1469	- 685	0.5626	0.6499	- 385	- 2024	- 2410
40	0	0	- 3000	- 985	0	335	469	1265	- 589	0.5258	0.6178	- 310	- 1646	- 1956
44	0	0	- 1900	- 882	0	300	412	1111	- 518	0.4914	0.5873	- 254	- 940	- 1194
48	0	0	- 2000	- 807	0	274	366	988	- 460	0.4593	0.5582	- 211	- 963	- 1175
52	0	0	- 1500	- 741	0	252	325	878	- 409	0.4292	0.5306	- 176	- 662	- 838
56	0	0	- 2600	- 650	0	221	278	749	- 349	0.4012	0.5044	- 140	- 1200	- 1340
60	0	0	- 2200	- 527	0	179	222	598	- 279	0.3749	0.4795	- 104	- 969	- 1073
64	0	0	- 1900	- 421	0	143	174	469	- 218	0.3504	0.4558	- 77	- 801	- 877
68	0	0	- 1600	- 327	0	111	133	359	- 167	0.3275	0.4333	- 55	- 645	- 700
72	0	0	- 1300	- 249	0	84	99	267	- 125	0.3060	0.4118	- 38	- 501	- 539
76	0	0	- 1100	- 182	0	62	71	192	- 89	0.2860	0.3915	- 26	- 406	- 432
80	0	0	- 900	- 124	0	42	48	129	- 60	0.2673	0.3721	- 16	- 319	- 335
84	0	0	- 700	- 77	0	26	29	79	- 37	0.2498	0.3537	- 9	- 238	- 248
88	0	0	- 400	- 44	0	15	16	44	- 21	0.2335	0.3362	- 5	- 130	- 134
92	0	0	- 300	- 22	0	8	8	22	- 10	0.2182	0.3196	- 2	- 93	- 96
96	0	0	- 100	- 10	0	3	4	9	- 4	0.2039	0.3038	- 1	- 29	- 30
100	0	0	- 100	- 3	0	1	1	3	- 1	0.1906	0.2888	- 0	- 29	- 29
TOTALS	103617	-13729	-100000	-100000	-30562	34000	13123	32152				46290	-46290	- 0

NOTE: Column (4) is calculated using the formulas in Table 4. Column (5) = -.25 * Tax Rate * (Premiums - Expenses for quarters 1 through 4). Column (6) = -1 * Tax Rate * Column (4). Column (10) is from Table 8, column (10). Column (12) = Tax Rate * (Column (10) + Column (11)). Columns (13) and (14), respectively, are computed at RF and RL, on the assumption that payments are made at the midpoint of each period. Quarterly discount rates are approximated as .25 * the annual rates RF and RL.

effects on both premiums and projected underwriting losses. The assumptions must be chosen with great care.

The MC example is presented in Table 9. β_L is assumed to be -.2 and the market risk premium is assumed to be 0.09. These assumptions result in a RADR for losses of 5.2 percent, retaining the 7 percent risk-free rate assumption from the NCCI example. The surplus commitment in Table 9 has been based on a surplus-to-reserves rather than a surplus-to-premiums ratio to facilitate comparison with the NCCI example. In both Tables 8 and 9 the insurer commits surplus equal to one-third of nominal expected losses or $33,333. An important difference between the two models is that the NCCI approach also recognizes a surplus commitment at time zero equal to the nominal after-tax underwriting loss. Thus, the initial surplus commitment is $40,008 in Table 8 (NCCI) and $33,333 in Table 9 (MC).

The MC model adopts the policyholder perspective, so that flows to and from the policyholder are discounted. The riskless flows, discounted at R_f, are premiums [column (1)], expenses [column (2)], the riskless part of the underwriting profits tax [column (5)], and the tax on investment income [column (12)]. Investment income is calculated as in the NCCI model, by multiplying the rate of investment return by the sum of the surplus and underwriting balance accounts. The risky flow, discounted at R_L, is the loss part of the underwriting profits tax [column (6)] and the loss itself [column (3)].

In the NCCI example (Table 8), the initial net cash flow is negative, reflecting the insurer's surplus commitment; and subsequent flows are positive. In the MC example, on the other hand, the early flows are positive, primarily reflecting premium payments, and the later flows are negative. This reflects the use of the policyholder rather than the insurer perspective in the MC model. Although the signs of the net cash flow streams in Tables 8 and 9 change only once, it is not unusual for the signs to change more than once in practical applications, especially in the NCCI case.

The relationships between premiums and the parameter values make sense intuitively. In Table 8, an increase in the premium leads to an increase in the IRR. Increasing the surplus-to-reserves ratio reduces the IRR, while an increase in the rate of investment return increases it. In Table 9, a decrease in the RADR (R_L) leads to an increase in the fair premium. This may initially appear counter-intuitive but actually makes sense. A negative value for β_L means that losses are negatively correlated with returns on the market portfolio so that underwriting profits are positively correlated with the market. Thus, accepting insurance policies increases the systematic risk of the company. An increase in (the absolute value of) β_L leads to a lower value of R_L. Losses then discount to a larger amount and thus the fair premium is higher.

SUMMARY AND CONCLUSIONS

The discounted cash flow approach is rapidly becoming predominant for property- liability insurance ratemaking. The two models that have achieved the most prominence are the MC and NCCI models. This article reviews and evaluates these two models and suggests modifications to improve their economic consistency. Formulas are provided for both models and for the underwriting and loss expense deductions under the Tax Reform Act of 1986. General principles and pitfalls of DCF ratemaking in insurance also are identified.

An important general principle is that ratemaking is prospective and each policy class should stand on its own. This implies that investment income credits should be based on investment returns expected to prevail during the policy runoff period, not on embedded yields. Such credits should reflect the insurer's actual investment strategy rather than the risk-free rate. Ratemaking should consider both the amount and timing of the cash flows associated with the policy class. Regulatory and accounting conventions are important only to the extent that they affect cash flows. Risk adjustments should reflect sound economic and financial principles; the analysis of book returns and variances carries no information that is useful in insurance ratemaking.

The choice between the MC and NCCI approaches ultimately must reflect the realities of the insurance market place. In a sense, both models represent sets of testable hypotheses about the way premiums would be set in competitive insurance markets. Both models involve assumptions about surplus flows. The assumptions are more tenuous here than in the usual capital budgeting examples because insurance equity is intermingled and committed to multiple lines of business and cohorts of policies. There are no unambiguous flows of capital to purchase tangible items like plant and equipment, with the latter devoted to specific projects. Rather, the models *assume* that surplus is committed at particular times and released according to a specified schedule. A priori reasoning can help in formulating arguments about which model is more reasonable. Ultimately, however, empirical testing will be needed to resolve the underlying issues.

One important unresolved issue is the appropriate level of surplus commitment. At the present time, there is no generally accepted theory of insurance surplus commitment. The tendency is to use market-wide ratios of surplus-to-reserves or surplus-to-premiums. However, overall ratios are unlikely to be appropriate for individual lines of insurance such as workers' compensation or automobile insurance. Recent research [e.g., Cummins (1988a) and Doherty and Garven (1986)] suggests promising approaches to the surplus commitment problem, but much work remains to be done. The timing of the commitment and the allocation of surplus among lines also would be fruitful areas for future research.

Additional research also is needed on other aspects of the models. For example, the payout tail is assumed to be known with certainty. If the payout pattern is subject to significant randomness or to drift, the fair premiums obtained from the present formulas will be incorrect. A synthesis of the DCF approach and the paid-loss reserve-development models presented in the international actuarial literature [e.g., Taylor (1986)] may provide a way to solve this problem.

Finally, more research is needed on the pricing of risk in insurance markets. Both the underwriting beta (β_L) and the cost of capital methodologies leave much to be desired. The underwriting beta is based on a single factor model that does not price the risk of ruin. Multiple factor models and/or models that price "non-systematic" risk should be developed and tested as alternatives. Because the estimation problems are severe [see Cummins and Harrington (1985)], part of the solution will involve a change in the way insurers collect and report data on premiums, losses, and other important cash flows. Companies should develop economically meaningful cash flow and profit data that are reported on a quarterly or monthly basis. This would obviate the need to use imperfect estimates of underwriting betas or industry-wide costs of capital that may not be applicable to individual lines of insurance or to individual firms.

Although significant research problems remain, one should not lose sight of the progress that has been made to date. The MC and NCCI models are soundly based in financial theory. The accounting methodologies and arbitrary profit loadings that were prevalent for many years are being abandoned. The insurance industry and its regulators have advanced significantly toward the recognition that cash flows, market values, and financial pricing techniques are necessary to arrive at fair premiums and rational underwriting decisions. The adoption of these concepts and the availability of the improved data needed for their application would bring needed stability to insurance markets.

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